A MEASUREMENT OF THE ROTATION RATE IN THE DEEP SOLAR INTERIOR

J. Schou[1], S. Tomczyk[2], and M.J. Thompson[3]

[1] HEPL Annex A201, Stanford University, Stanford, CA 94305-4085 USA; schou@solar.stanford.edu
[2] High Altitude Observatory, NCAR, P.O. Box 3000, Boulder, CO 80307-3000, USA; tomczyk@hao.ucar.edu
[3] Astronomy Unit, Queen Mary and Westfield College, Mile End Road, London E1 4NS, U.K.; mthompson@solar.stanford.edu

ABSTRACT

We present a measurement of the rotation rate in the interior of the Sun based on six months of observations with the LOWL instrument. LOWL allows observation of oscillations with degrees from 0 to $\sim$ 80, thus providing a homogeneous low- and intermediate-degree dataset. Moreover, it is able to make spatially resolved observations of low degree modes, thereby making it possible to separate spatially the different modes within a given multiplet. This reduces the potential for systematic errors compared to observations using integrated sunlight. We have used observations of the frequency splittings of modes with degrees from 1 to 80 to infer the rotation rate in the solar interior down to $\sim 0.2R_\odot$ with some radial resolution and without excessive errors. Over part of the range we have also been able to estimate the latitudinal variation of the rotation rate. We confirm earlier findings that near the base of the convection zone the solar rotation profile undergoes a transition from surface-like differential rotation to a rotation rate that is independent of latitude. Additionally, we find that from the base of the convection zone down to 0.2 $R_\odot$ our measurement is consistent with rigid body rotation at a rate somewhat lower than the surface equatorial rate. This measurement provides strong constraints on the theories of angular momentum transport in solar-type stars.

Keywords: solar oscillations, solar internal rotation.

1. BACKGROUND

Helioseismic measurements of intermediate-degree frequency splittings by various observers (Duvall, Harvey and Pomerantz, 1986; Christensen-Dalsgaard and Schou, 1988; Brown *et al.*, 1989; Rhodes *et al.*, 1990) indicate that throughout the convection zone the solar angular velocity follows a surface-like latitude distribution with a transition to latitude independent rotation near the base of the convective envelope. Below $\sim 0.4R_\odot$, however, the solar rotation rate remains highly uncertain due to a lack of accurately measured splittings for low-degree modes. Attempts to measure these splittings using spatially resolved observations have generally failed due to instrumental noise. Observations of the Sun in integrated light have shown the requisite stability to observe low-degree oscillations. However, these observations are unable to discriminate spatially between individual oscillation modes, and therefore require modeling of the blended modes to obtain the splittings.

This paper presents results from the LOWL instrument, which observes both low- and intermediate-degree oscillations with spatial resolution thus providing a reliable estimate of solar internal rotation between 0.2 and $0.85R_\odot$.

2. OBSERVATIONS

The observations used in this study were obtained with the recently completed LOWL instrument (see Tomczyk *et al.*, 1995a for more details). The LOWL is a velocity imaging instrument based on a Potassium Magneto-Optical-Filter (MOF). It obtains full disk images simultaneously in two 2 pm bandpasses displaced $\pm$110 pm into opposite wings of the solar absorption line of Potassium at 769.9 nm. Images are obtained every 15 seconds with a spatial resolution of 25 arcsec and a noise level of 15 m/s/pixel. The radial velocity v is estimated at each pixel from $(I_b - I_r)/(I_b + I_r)$, where I_b and I_r are the intensities through the blue and red bandpasses. For the present study the velocity images were averaged over 1 minute intervals. The atomic vapors employed in the MOF make the instrument extremely stable, allowing solar noise-limited observations down to degree l=0.

The instrument is located at an altitude of 3414 m on Mauna Loa on the island of Hawaii and became operational on Feb 26, 1994. The observations used for this study were collected over a 6 month period between Feb 26, 1994 and Aug 26, 1994 with a duty cycle of approximately 20%.

3. ANALYSIS

The method by which mode frequencies are obtained from the averaged velocity images is described in Tomczyk *et al.* (1995b). Since it is difficult to estimate frequencies for individual m's, the variation of mode frequency ω_{nlm} with azimuthal order m was

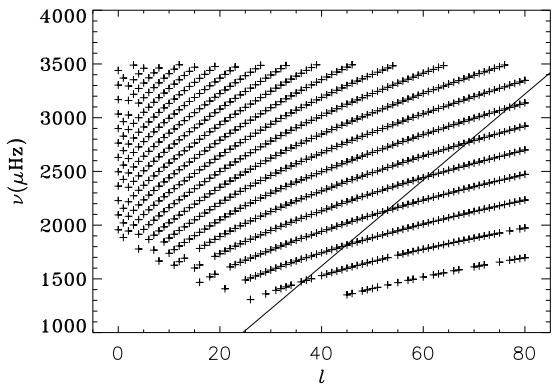

Figure 1: $l-\nu$ diagram showing the modes used for this paper. The solid line indicates $\nu/L = 40\mu Hz$.

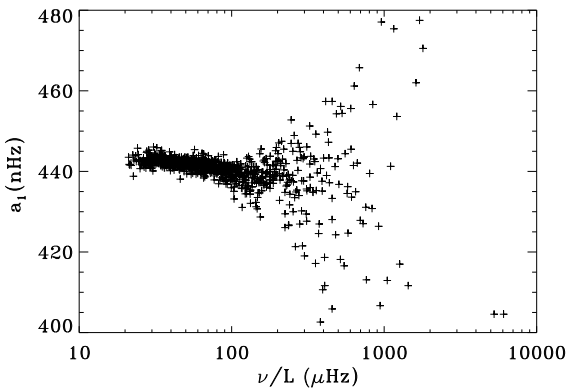

Figure 2: a_1 as a function of ν/L. Some splittings are outside the range used for this plot.

expanded using so-called a coefficients:

$$\omega_{nlm}/2\pi = \nu_{nl} + \sum_{j=1}^{j_{\max}} a_j(n,l) P_j^{(l)}(m), \qquad (1)$$

where ν_{nl} is the average mode frequency, the a_j are the fitted coefficients and the $P_j^{(l)}$ are a set of orthogonal functions of degree j (cf. Schou *et al.* 1994). The set of modes fitted is shown in Fig. 1. Figure 2 shows a_1 which describes the linear variation of mode frequency with m and relates to the rotation rate, as a function of ν/L (where $L = \sqrt{l(l+1)}$) which is related to the turning point depth of the mode.

The odd a coefficients are related to the rotation rate Ω at radius r and colatitude θ by

$$2\pi\, a_{2s+1}(n,l) \;=\; \int_{r,\theta} K_{nls}^{(a)}(r,\theta)\, \Omega(r,\theta)\, r\, \mathrm{d}r\mathrm{d}\theta + \epsilon_{nls}\ , \qquad (2)$$

(Schou *et al.* 1994) where the kernels $K_{nls}^{(a)}(r,\theta)$ are functions of the equilibrium structure of the Sun, and ϵ_{nls} represent data errors. The errors in the coefficients are assumed to be independent and normally distributed, with zero mean and standard deviation $\sigma_{nls}^{(a)}$.

The North-South symmetric component of the internal rotation rate may be estimated by inverting the constraints (2). We have employed a 2-D Regularized Least Squares (RLS) inversion method which minimizes

$$\sum_{nls} \left[\frac{\int_{r,\theta} K_{nls}^{(a)} \bar{\Omega} r \mathrm{d}r\mathrm{d}\theta - 2\pi a_{2s+1}(n,l)}{\sigma_{nls}^{(a)}} \right]^2$$
$$+\mu_r \int_{r,\theta} f_r(r,\theta) \left(\frac{\partial^2 \bar{\Omega}}{\partial r^2} \right)^2 \mathrm{d}r\mathrm{d}\theta$$
$$+\mu_\theta \int_{r,\theta} f_\theta(r,\theta) \left(\frac{\partial^2 \bar{\Omega}}{\partial \theta^2} \right)^2 \mathrm{d}r\mathrm{d}\theta\ . \qquad (3)$$

Here $\bar{\Omega}(r,\theta)$ is the solution (an estimate of the true underlying rotation rate), which we represent as a bilinear function on a discretized mesh in r and θ. The functions f_r and f_θ are non-negative weights (see Schou *et al.* 1994). The choice of values for μ_r and μ_θ determines the trade-off between resolution and noise amplification.

The solution at (r_0,θ_0) may be represented as a linear combination of the data, with some weights $c_{nls}^{(a)}(r_0,\theta_0)$:

$$\bar{\Omega}(r_0,\theta_0) \;=\; \sum_{nls} c_{nls}^{(a)}(r_0,\theta_0)\, a_{2s+1}(n,l)\ . \qquad (4)$$

An estimate of the random error at a given point can be estimated as

$$\sigma(r_0,\theta_0)^2 = \sum_{nls} \left[c_{nls}^{(a)}(r_0,\theta_0) \sigma_{nls}^{(a)} \right]^2 . \qquad (7)$$

This is only an estimate of the random error as propagated from the input splittings. The difference between the inferred and the true rotation rate could be much larger due to unresolved features in the true rotation rate.

4. RESULTS

Our dataset comprised 2119 a_1, a_3 and a_5 coefficients for 716 (n,l) multiplets with l ranging from 1 to 80, and with $\nu/L > 40\mu$Hz. The attainable resolution in the radial direction, as measured from the extent of the averaging kernels, is generally better than that in the latitudinal direction. This could be improved considerably in the outer parts of the Sun if one were to have more a coefficients per multiplet; such an analysis is in progress.

The inferred rotation rate and $1-\sigma$ uncertainties are shown in Figures 3a and 3b respectively. Figure 4 shows cuts through the inferred rotation rate at colatitudes $\theta = 0°$, $45°$ and $90°$. In the convection zone we see a transition from surface-like differential rotation to a rotation rate that is independent of latitude, in basic agreement with previous findings. To within the resolution of our inversion, the differential rotation vanishes at a point consistent with recent helioseismological estimates of the base of the

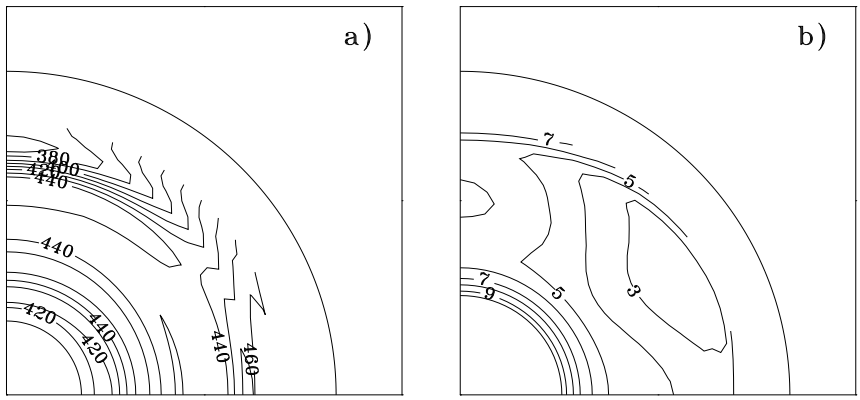

Figure 3: (a) The solar rotation rate between $r = 0.2R_\odot$ and $r = 0.85R_\odot$, as inferred using the 2-D RLS technique with $\mu_r = 10^{-7}$, $\mu_\theta = 10^{-3}$. The contour spacing is 10 nHz. (b) The corresponding 1-σ error levels (values are in nHz).

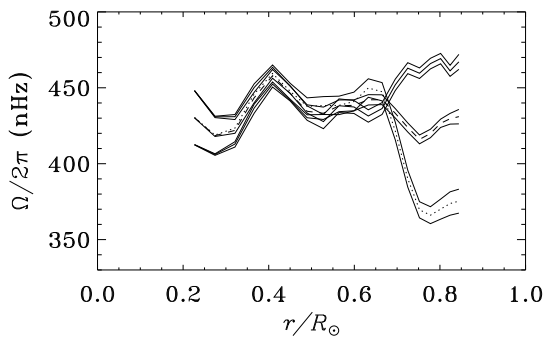

Figure 4: The inferred solar rotation rate as in Fig. 3a, at the equator (solid line), at colatitude 45° (dashed line) and at the pole (dotted line). One-sigma error levels are indicated by the thin solid lines.

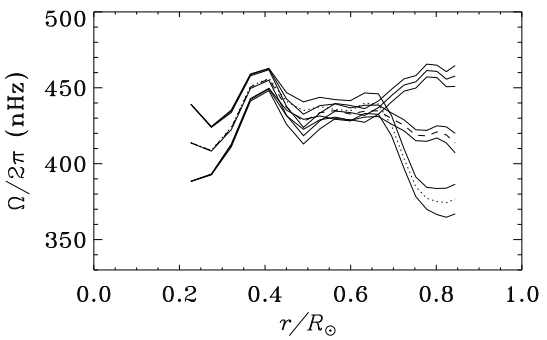

Figure 5: As Fig. 4, but for only the first three months of LOWL observations (from Tomczyk *et al.* 1995b). The line styles are as in Fig. 4.

convection zone ($\approx 0.72R_\odot$).Below $r \approx 0.65R_\odot$ we find no significant variation of the rotation rate with radius, except possibly for the increase at $r \approx 0.4R_\odot$. Statistical uncertainties in the current data prevent the reliable estimation of internal rotation at depths below $r = 0.2R_\odot$. As we discuss below, these uncertainties will decrease with the acquisition of longer datasets.

In $r \lesssim 0.4R_\odot$ we see no variation with latitude. This is simply a reflection of the fact that below this depth we cannot localize kernels at different latitudes (see Tomczyk *et al.* 1995b) and is a fundamental limitation imposed by the fact that the deepest interior is only sampled by low-degree p-modes, for which there are few different m values. On the other hand, further out (e.g. at $r_0 = 0.5R_\odot$) we can achieve some latitudinal resolution. If there were substantial latitudinal variation at $r_0 = 0.5R_\odot$ we should be able to detect it. As can be seen from Figure 4, however, we see no significant variation with latitude anywhere in the radiative interior.

For comparison, we also show (Fig. 5) the corresponding results obtained by Tomczyk *et al.* (1995b) from just the first three months of our 6 months of observations. We note that the error bars on our new inversion are smaller than when we had only three months of observations. One would indeed expect the data errors, and hence the error bars on the inversion if the resolution is unchanged, to decrease roughly as the inverse square-root of the measurement duration. We note too that the same features found by Tomczyk *et al.* in the deep interior are also present in the inversion of the 6 month set. In particular, the local maximum at $r \approx 0.4R_\odot$ is present in both inversions. We are in the process of analyzing the second three months separately to be able to compare the two independent three month sets. In fact, the only differences between Figs. 4 and 5 that may appear to be significant are in the convection zone: these are probably artifacts of our reduction and inversion, but need to be investigated further.

5. IMPLICATIONS FOR THE PHYSICS OF THE DEEP INTERIOR

Solar models including only hydrodynamical mechanisms for angular momentum transport indicate that

the presence of molecular weight gradients can inhibit the transfer of angular momentum out of the solar core (Pinsonneault *et al.*, 1989). These models indicate that the Sun can retain a large fraction of its initial angular momentum at depths $\lesssim 0.3 R_\odot$ where these gradients become significant. While these models predict a rotation rate at 0.2 $R_\odot$ between approximately 10 and 30 times the surface equatorial rate, at depths $>0.4 R_\odot$ the predicted rotation rate does not differ substantially from the surface rate. On the other hand, it has been suggested that a reasonably small (about 10^{-2} G) poloidal field in the solar radiative zone is sufficient to impose rigid rotation (Rosner and Weiss, 1985; Mestel and Weiss, 1987). The plausibility of this argument has been demonstrated by numerical simulations (Charbonneau and MacGregor, 1993). Past frequency splitting measurements have provided meaningful constraints on the rotation over only the outer half of the Sun and therefore have been unable to discriminate between these vastly different scenarios (Demarque & Guenther, 1988).

Our results indicate that the rather uniform rotation detected in the outer radiative interior extends down at least to $0.2 R_\odot$. This measurement argues strongly against theories of solar angular momentum transport which predict large radial gradients in the solar interior angular velocity.

REFERENCES

1. Brown, T.M. Christensen-Dalsgaard, J., Dziembowski, W.A., Goode, P., Gough, D.O. and Morrow, C.A., 1989, *ApJ*, **343**, 526.
2. Charbonneau, P. and MacGregor, K.B., 1993, *ApJ*, **417**, 762.
3. Christensen-Dalsgaard, J., and Schou, J., 1988, in *Seismology of the Sun and Sun-like Stars*, ESA Publications 286, Noordwijk, (E.J. Rolfe, ed.), 149.
4. Demarque, P. and Guenther, D.B., 1988, in *Seismology of the Sun and Sun-like Stars*, ESA Publications 286, Noordwijk, (E.J. Rolfe, ed.), 99.
5. Duvall, T.L., Harvey, J.W. and Pomerantz, M., 1986, *Nature*, **321**, 500.
6. Mestel, L. and Weiss, N.O. 1987, *MNRAS*, **226**, 123.
7. Pinsonneault, M.H., Kawaler, S.D., Sofia, S., and Demarque, P., 1989, *ApJ*, **338**, 424.
8. Rhodes, Jr., E. J. , Cacciani, A., Korzennik, S.G., Tomczyk, S., Ulrich, R.K. and Woodard, M.F., 1990, *ApJ*, **351**, 687.
9. Rosner, R. and Weiss, N.O., 1985, *Nature*, **317**, 790.
10. Schou, J., Christensen-Dalsgaard, J. and Thompson, M.J., 1994, *ApJ*, **433**, 389.
11. Tomczyk, S., Streander, K., Card, G., Elmore, D., Hull, H. and Cacciani, A., 1995a, *Sol. Phys.*, in press.
12. Tomczyk, S., Schou, J., Thompson, J., 1995b, *ApJ Lett.*, submitted.