THE SENSITIVITY OF VARIOUS MODE SETS FOR PROBING DIFFERENTIAL ROTATION SHEAR ZONES

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ABSTRACT

The potential of forthcoming datasets from GONG and SOI for resolving long-lived azimuthal jets and shearing flows is investigated. We construct various artificial datasets, containing noise resembling that for a one-year observing run. These are then inverted, using a 2-D Regularized Least Squares inversion. In particular, we investigate the ability of the RLS method to form well-localized averages of the rotation rate, as measured by the averaging kernels, using an extensive mode set as well as subsets thereof. We show that it is possible to keep the noise in the solution down to a few nanohertz in much of the solar interior, while obtaining very reasonable resolution, for a GONG-like dataset. At low latitudes in the middle of the convection zone, for example, we achieve an angular resolution of less than ten degrees, and a radial resolution of about 0.04 solar radii, which we find very encouraging. The averaging kernels depend in detail upon the mode set, however. In particular, a reduction in the number of modes tends to introduce small-scale near-surface structure in the averaging kernels which would adversely affect the inferred rotation rate, if solar rotation were to display sharp features near the surface.

1. INTRODUCTION

In eager anticipation of the large sets of high-quality data that GONG and SOI will provide, we have performed experiments with artificial data to test our ability to detect long-lived azimuthal jets and shearing flows in the solar interior. We have constructed a fictitious internal rotation profile containing jetlike features in the convection zone and outer part of the radiative interior, and a shear layer near the base of the convective envelope. We have used this profile to compute frequency splittings for all individual (n, l, m) modes (including f modes) in the frequency range 1–5 mHz, and degrees $1 \le l \le 200$. In all, our dataset contains nearly 200 000 splittings. We do not go quite as high in l as the 1992 GONG Hare and Hounds experiment (Gough & Toomre 1993) but nonetheless we have nearly three times as many splittings because we include all l values up to our maximum value $l = 200$.

Although we have not added a noise realization in the experiments presented here, we have assumed uncertainties in the data similar to those expected from an observation run of one year in duration.

We have also performed some experiments with subsets of this mode set, to test the sensitivity to different selections of modes.

2. INVERSION METHOD

As is well known, the splitting data are related to the underlying rotation rate $\Omega(r,\theta)$ by kernels $K_{nlm}(r,\theta)$ according to:

$$2\pi\,\Delta\nu_{nlm} \;\equiv\; \Delta\omega_{nlm} \;=\; \int_{r,\theta} K_{nlm}\,\Omega(r,\theta)\,r\,\mathrm{d}r\mathrm{d}\theta\ . \tag{1}$$

Artificial data were calculated thus for our test problem. These data were then inverted using a 2-D Regularized Least Squares inversion (Schou *et al.* 1994) in which one minimizes

$$\sum_{nlm}\left(\frac{\int_{r,\theta} K_{nlm}\bar\Omega r\mathrm{d}r\mathrm{d}\theta - \Delta\omega_{nlm}}{\sigma_{nlm}}\right)^2 + \mu_r\int_{r,\theta} f_r(r,\theta)\left(\frac{\partial^2\bar\Omega}{\partial r^2}\right)^2\mathrm{d}\theta\,\mathrm{d}r + \mu_\theta\int_{r,\theta} f_\theta(r,\theta)\left(\frac{\partial^2\bar\Omega}{\partial\theta^2}\right)^2\mathrm{d}\theta\,\mathrm{d}r\ . \tag{2}$$

Here $\bar\Omega(r,\theta)$ is the solution, which we represent as a bilinear function on a mesh comprising 24 uniformly spaced intervals in θ and 50 intervals in r spaced approximately uniformly in the acoustical radius. As is often done, we here regularize the problem by penalizing solutions which have large second

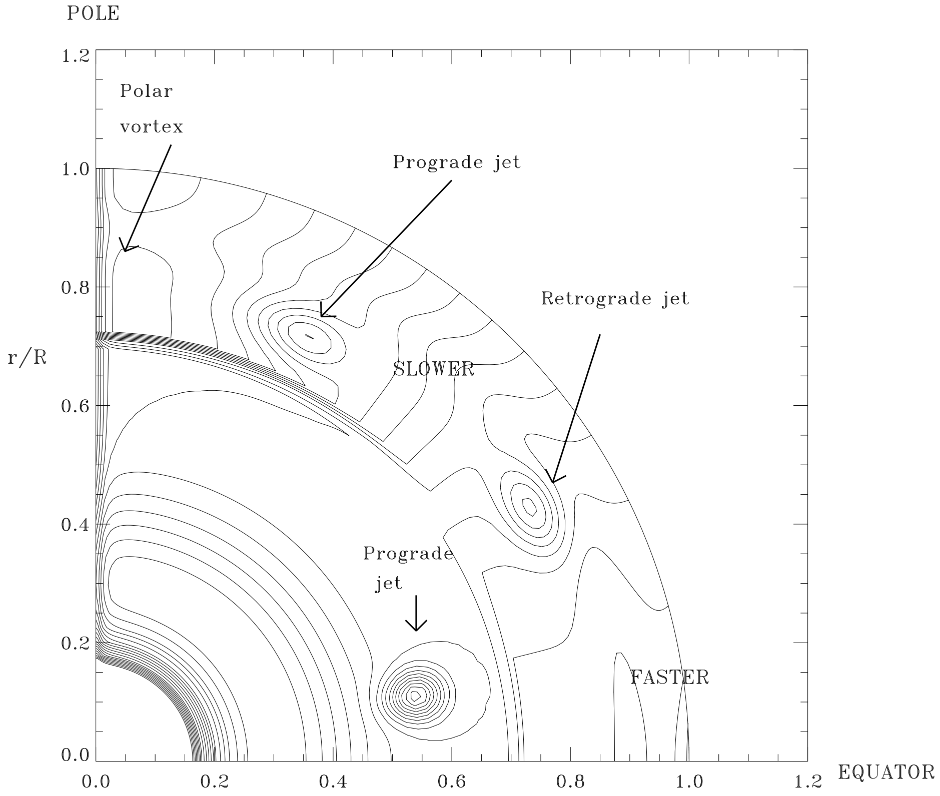

Figure 1: Input rotation profile used for generating artificial splitting data for our experiments. The contour spacing is 10 nHz.

derivatives. The functions f_r and f_θ are non-negative weights that determine the relative importance of the smoothing constraints at different locations: they were chosen to be the same as in Schou *et al.* (1994). There is a trade-off to be made between improving the resolution and reducing the effect of data errors on the solution: this is determined by the values of the parameters μ_r and μ_θ.

3. RESULTS WITH THE FULL MODESET

As a test problem, we have taken a solar model and computed (using first-order perturbation theory) the frequency splittings $\Delta\nu_{nlm} \equiv \nu_{nlm} - \nu_{nl0}$ that would be caused by the fictitious steady, axisymmetric rotation profile illustrated in Fig. 1.

Inverting these data with the full mode set, and with $\mu_r = 10^{-7}$, $\mu_\theta = 10^{-5}$, gives the solution shown in Fig. 2a. The contour spacing is 10 nHz. The corresponding standard errors, which indicate the effect that noise would have on the solution were the data to have errors corresponding to one year's observations, are shown in Fig. 2b. Except at high latitudes and deep in the interior, the error levels are generally below the contour spacing used in panel (a). Thus in much of the star the topology of the solution contours would not have been much changed had the data contained noise. Indeed the main effect of noise in that region is simply to make the contours slightly jagged. We are encouraged that with this dataset and with a suitable choice of values for the trade-off parameters, we have been able to keep the sensitivity of the solution to noise so small, whilst resolving quite unambiguously the three jets. The fast core is also resolved fairly well, though the polar vortex cannot be seen. This is because relatively few modes are sensitive to conditions at high latitudes, and indeed all the rotation kernels vanish at the pole itself ($\theta = 0$).

The resolution that has been attained can be judged from the extent to which small-scale features in the input rotation profile have (or have not) been smeared out in the solution. Of course, one can get a more detailed picture of the resolution by inspecting averaging kernels. The poorer the resolution, the more smeared out the features and the wider the main peak of the averaging kernels. We have exhibited kernels corresponding to this mode set and for similar trade-off parameters elsewhere (Christensen-Dalsgaard *et al.* 1995a). At low latitudes in the middle of the convection zone, for example, we achieve an angular resolution of less than 10°, and a radial resolution of about $0.04R$.

4. TESTS WITH SUBSETS OF MODES

For a variety of reasons, one might perform inversions similar to these but with a subset of the modes we

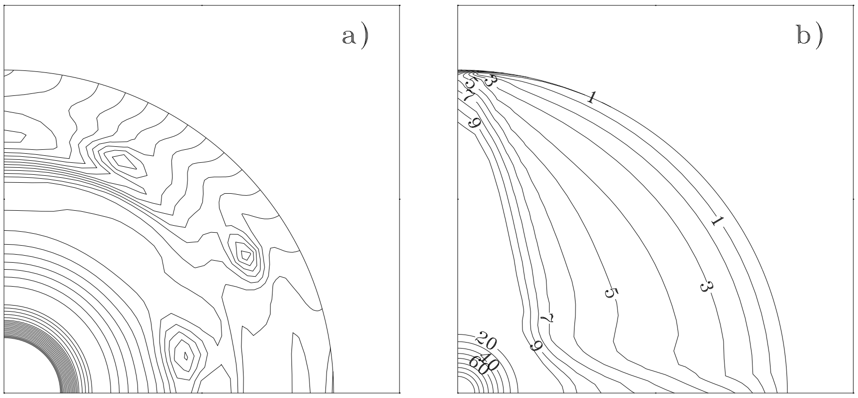

Figure 2: (a) The reconstructed rotation profile obtained with our 2-D regularized least squares inversion, as described in the text, using the full set of nearly 200,000 splittings. The contour spacing is 10 nHz. The data were noise-free, but data uncertainties corresponding to one year's observations were assumed. (b) The standard errors corresponding to the solution in panel (a), in nHz.

have used, for example because the computation with the full mode set would take longer than one wishes to wait, or because one simply cannot obtain data for the full modeset (as a result, for example, of limited telemetry). We have therefore tried inverting, with the same trade-off parameters, three subsets of our original mode set, in each case including all modes up to $l = 30$:

• **Set A:** all modes up to $l = 30$; all n and m at every tenth l thereafter

• **Set B:** all modes up to $l = 30$; a random selection in (n, l, m) of 10% of the modes for $l > 30$

• **Set C:** all modes up to $l = 30$; a random selection in (l, m), including all n's for each pair selected, for a total of 10% of the modes for $l > 30$.

The number of splittings in each subset is 28,210, 27,523 and 27,975 respectively. (The different numbers are statistical fluctuations in our random selection of modes: we do not believe that the differences have any significant effect on the results below.)

A mathematically trivial effect is that similar resolution to before is achieved only at the expense of larger errors. (Roughly speaking, having only half the modes, for example, will result in the noise level increasing by $\sqrt{2}$.) Apart from this, the solutions for our test problem with the three subsets are almost indistinguishable from the solution obtained for the full set. However, the insensitivity of the solution to the mode set is largely a consequence of our artificial rotation profile being rather smooth, as we now discuss.

The averaging kernels for the different subsets differ in detail, as illustrated in Fig. 3. By way of example, we have shown kernels for each subset for target location $r_0 = 0.9R$ and target latitudes $\theta_0 = 90°$, $45°$ and $0°$. As is typical for Regularized Least Squares inversions of helioseismic data, the averaging kernels have some unwanted structure near the surface. The extent of this near-surface structure differs between the three subsets, being least severe for Set A and worst for Set B at all latitudes. Since the averaging kernels have somewhat different structure near the surface in the three cases, the solutions would have differed more appreciably, if the rotation had varied on small scales in that region. In the case of 1-D rotational inversions, we have previously found that weeding out say half the modes at random makes little appreciable effect to the quality of the averaging kernels. In the present 2-D case, taking modes completely at random in (n, l, m) is clearly not the optimal way of culling a large modeset, if one has the freedom to choose the subset for inversion. We speculate that, if one is forced to use only a subset of individual splittings, it is advantageous in the 2-D inversion to have all m values for given n and l, perhaps because of their completeness in latitude (as indeed Douglas Gough suggested to us before we performed this experiment!).

Since it can be difficult to fit individual mode frequencies, the variation of the mode frequencies with m has traditionally been expanded using so-called a coefficients:

$$\nu_{nlm} = \nu_{nl} + \sum_{j=1}^{j_{\max}} a_j(n,l) P_j^{(l)}(m) \; , \qquad (3)$$

where ν_{nl} is the average mode frequency, the a_j are the fitted coefficients and the $P_j^{(l)}$ are a set of orthogonal polynomials of degree j (cf. Schou *et al.* 1994). We have therefore performed experiments

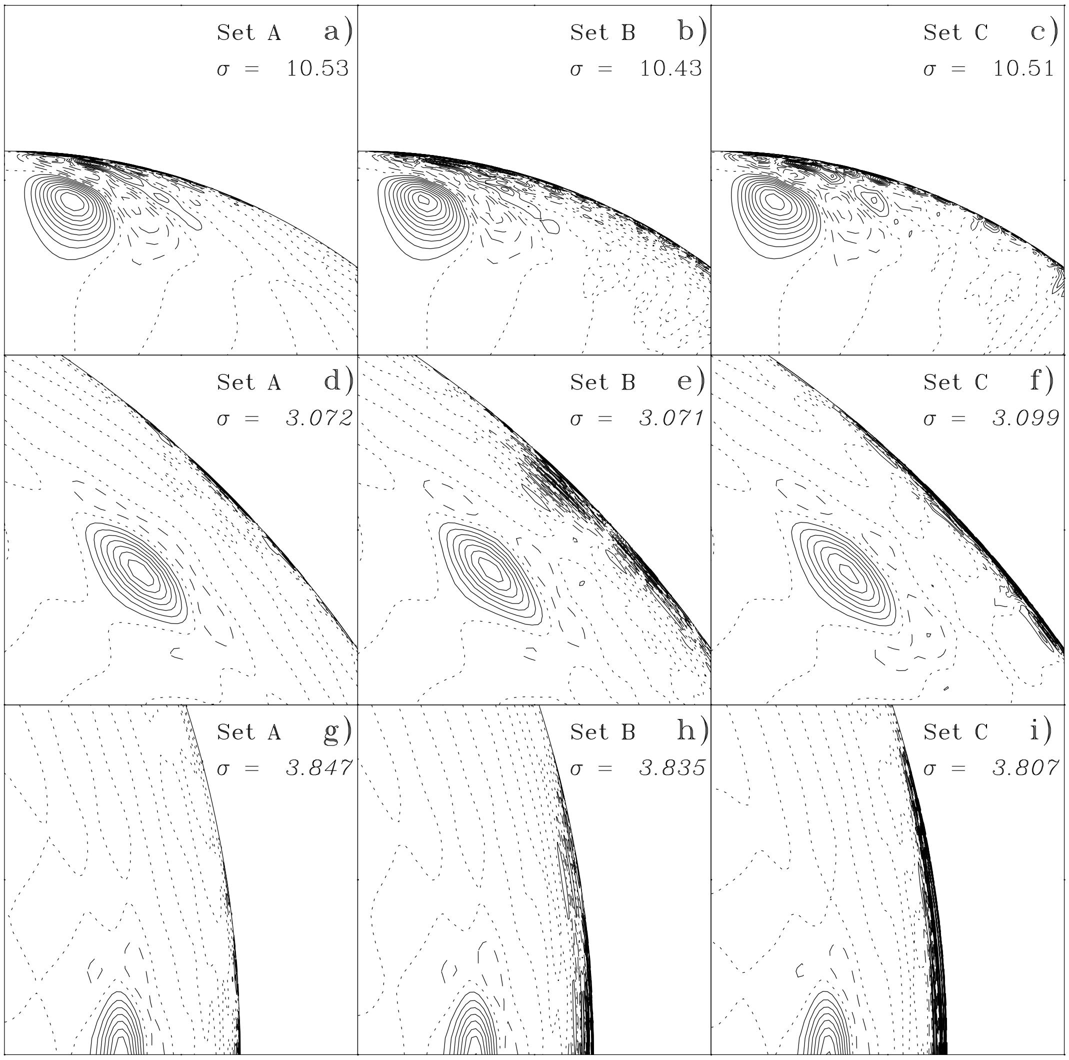

Figure 3: Close-ups of averaging kernels targetted at radius $0.9R$ and latitude $90°$ (panels a-c), latitude $45°$ (panels d-f) and latitude $0°$ (panels g-i). The first column is for Set A, the second for Set B and the third is for Set C. The corresponding $1-\sigma$ standard errors in the solutions are indicated. The field of view is $0.6R$ on the side for the top row, and $0.3R$ on the side for the second and third rows. The contour levels are different in the three rows, since they were chosen so as to be roughly the same as a fraction of the height of the main peak: they are however the same for each panel within a given row. Positive contours are drawn with a solid curve, negative contours with a dashed curve, and the zero contour is shown as a dotted curve.

where we replace individual splittings with a coefficients at low (and high) frequency. These experiments have indicated that the inversions are pretty robust (Christensen-Dalsgaard *et al.* 1995b). Thus it does not appear to be a significant disadvantage to lose complete coverage in m provided one has complete coverage (or at least a large number of a coefficients) over a substantial fraction of the (n, l) domain.

5. CONCLUSIONS

The promising results of our present experiments, together with the possible use of complementary techniques for detecting narrow features or performing local analyses, indicate that there is great scope for detecting interesting features in Sun's flow field with the GONG data that will shortly be available. If we have to work with only a subset of modes, it appears that we should avoid partial, random coverage in m.

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