ROTATION OF THE SOLAR CORE

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ABSTRACT

We present inferences for the solar core rotation derived from BiSON and LOWL data on frequency splittings. The LOWL data are for the two-year period Feb. 1994 – 1996. In the combined dataset we use LOWL splitting coefficients (a_1, a_3, a_5) for $l > 2$ and BiSON splittings for $l = 1$ and $l = 2$ for an overlapping period. We have employed a new two-dimensional implementation of the optimally localized averages (OLA) inversion method. As in one dimension, the OLA produces better-localized averaging kernels than do least-squares techniques. This is particularly beneficial in the deep interior, where it is in any case difficult to localize a kernel. With the OLA technique and BiSON and LOWL data we are able to obtain averaging kernels confined to the solar core, thus giving a very clean measure of the rotation rate of the Sun's deep interior.

1. MOTIVATION

The angular velocity of the solar core has been the subject of considerable debate. In particular, recent independent measurements by the IRIS and BiSON networks have given apparently conflicting results. The IRIS data indicate that the core rotates at, or perhaps slightly above, the surface rate (*e.g.* Lazrek *et al.* 1996), whereas the BiSON data have shown evidence for core rotation *below* the mean rate of the solar envelope (Elsworth *et al.* 1995).

Inferences about the core rotation are complicated by the fact that the dominant contributions to the frequency splittings come from the outer parts of the Sun. Thus in the analysis data for the low-degree modes which penetrate to the core must be combined with data for higher-degree modes, to eliminate the contributions from the rest of the Sun. Ideally, the analysis to determine the core rotation should include directly the results for the higher-degree modes. Unlike Elsworth *et al.* (1995), who used a simplified representation for the rotation of the solar envelope, we have carried out an inversion which includes sufficient information about the envelope rotation to allow the construction of rotation measures predominantly localized to the core.

2. INVERSION TECHNIQUES

The inverse problem for the solar internal angular velocity $\Omega(r,\theta)$ is

$$\int_0^R \int_0^{\pi/2} K_i(r,\theta)\Omega(r,\theta) r\mathrm{d}r\mathrm{d}\theta = d_i \; , i = 1, \ldots, M \; , \quad (1)$$

where r is distance to the centre and θ is colatitude. Here the index i numbers the data d_i. These may be either individual splittings $m^{-1}\Delta\omega_{nlm}$ for a mode of radial order n, degree l and azimuthal order m, or expansion coefficients $2\pi a_k(l,m)$ resulting from suitable polynomial expansions of $\Delta\omega_{nlm}$. We assume that the kernels K_i, which obviously depend on the type of data considered, are known.

The observed data are acoustic modes of degree $l = 1 - 99$. Our ability to infer the rotation of the deep solar interior depends on the availability of data for deeply penetrating modes. The behaviour of the kernels in the core of the Sun depends on two properties: the modes have a lower turning point at $r = r_t$, such that $c(r_t)/r_t \simeq \omega/(l+1/2)$, c being the adiabatic sound speed, and thus modes of low degree and/or high frequency penetrate most deeply; and, near the centre, the kernels behave like $K_i \propto r^{2l}$. However, for $l = 1$ the leading-order terms cancel, so that $K_i \propto r^4$. Hence splittings for $l = 1$ are less sensitive to the core rotation than might otherwise have been expected.

The inversions were carried out using the two-dimensional regularized least-squares (RLS) technique described by Schou *et al.* (1994) and by two-dimensional generalizations of the multiplicative (MOLA) and subtractive (SOLA) optimally localized averages techniques used extensively in the one-dimensional case (e.g. Gough 1985; Pijpers & Thompson 1992). The latter techniques will be more fully described elsewhere; for MOLA a weight function was chosen such as to localize as far as possible the averaging kernels to the central region of the Sun, whereas for SOLA we used target functions of Gaussian shape in radius and latitude.

3. DATA

The BiSON $l = 1$ and $l = 2$ splitting data were derived from the analysis of several 8 and 16-month spectra, generated from Doppler velocity residuals collected between 1991 and 1995, and analysed as described by Chaplin *et al.* (1996).

For $3 \leq l \leq 99$ we used splittings (in coefficient form) from the LOWL instrument (Tomczyk *et al.* 1995) taken from two years of observations covering the period Feb. 26, 1994 to Feb. 25, 1996, and fitted as described by Schou (1992). Due to the rapid increase in linewidth with frequency, only modes with $\nu \leq 3500\mu$Hz have been used.

Figure 1 illustrates the data for modes of degree $l = 1 - 6$. There is no obvious inconsistency between the BiSON

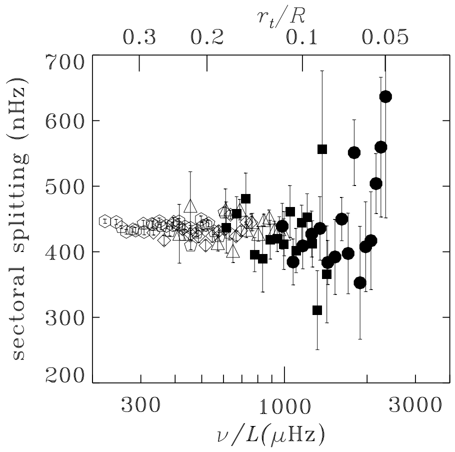

Figure 1: Sectoral splittings for BiSON $l = 1$ (filled circles) and $l = 2$ (filled squares) and LOWL $l = 3$ (triangles), $l = 4$ (diamonds), $l = 5$ (pentagons), and $l = 6$ (hexagons), plotted as a function of ν/L with turning point radius on the upper axis.

data (for $l = 1, 2$) and the LOWL data (for $l = 3, 4$). However, it is evident that the error increases strongly with increasing frequency and decreasing degree, corresponding, unfortunately, to more deeply penetrating modes.

4. RESULTS

The principal results of the inversions are shown in Fig. 2 which combines SOLA results for $r > 0.3R$, RLS results and MOLA results in the core. The figure shows the now familiar pattern of convection-zone rotation. The apparent disagreement between the SOLA and RLS results in the envelope arises because of differences in the detail of the averaging kernels for the different methods. The results confirm the downturn in the rotation in the core inferred by Elsworth *et al.* (1995), at least qualitatively.

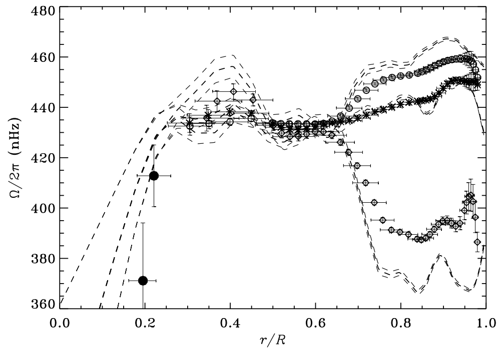

Figure 2: Radial cuts at 0, 30, and 60 degrees latitude through SOLA (open diamonds) and RLS (dashed lines) rotation inversions for the LOWL +BiSON data. The MOLA results for the core are shown as solid circles.

To illustrate our ability to obtain information about the core rotation, Fig. 3 shows sample averaging kernels for the MOLA and RLS inversions. The RLS kernel is rather poorly localized, with a structure indicating that the inferred solution at the target radius of $0.23R$ to some extent corresponds to a linear extrapolation from the solution at greater radii; this is a natural consequence of the assumed regularization which minimizes the second derivative of the solution, and is also evident from the solution shown in Fig. 2. Furthermore, the kernel displays unwanted structure close to the surface. Choosing a smaller target radius for the RLS has no significant effect on the averaging kernel. In contrast, the MOLA technique has evidently succeeded in confining $\mathcal{K}$ to $r \lesssim 0.25R$, with only very small contributions from the rest of the Sun. Thus to this extent the apparent low values of the core rotation shown in Fig. 2 represent a localized average over the core.

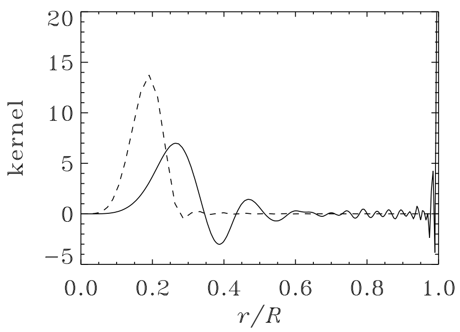

Figure 3: Radial slices at the equator through OLA averaging kernel targeted at $r = 0$ (dashed) and RLS averaging kernel targeted at $r = 0.23R$ (solid).

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