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Low-Degree Frequency Splitting Measurements and the Rotation Rate of the Solar Core

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Abstract. We summarize recent measurements of the frequency splitting of low-degree p-modes. We discuss systematic errors associated with these measurements and argue that the inconsistent ℓ=1 measurements are due to systematic errors. Also, we present detailed observations from the LOWL instrument and two-dimensional rotation profiles of solar internal rotation derived from these data. We argue that the current frequency splitting measurements, taken as a group, are consistent with solid body rotation of the solar interior below the base of the convection zone.

Key words: Sun: internal rotation - Sun: oscillations

1. Introduction

Observation of the frequency splitting of low-degree acoustic oscillations allows the inference of the rotation rate of the solar core. This important measurement is made difficult by there being few low-degree modes available for measurement, while the sensitivity of the p-mode frequencies to structure and rotation decreases with depth into the sun. Inferred core rotation rates are very sensitive to small random and systematic errors inherent in the low-degree frequency splitting measurements. If all of the past observations and their errors are taken at face value, one is forced to conclude that the solar core rotation rate is highly variable on short time scales. The alternative explanation is

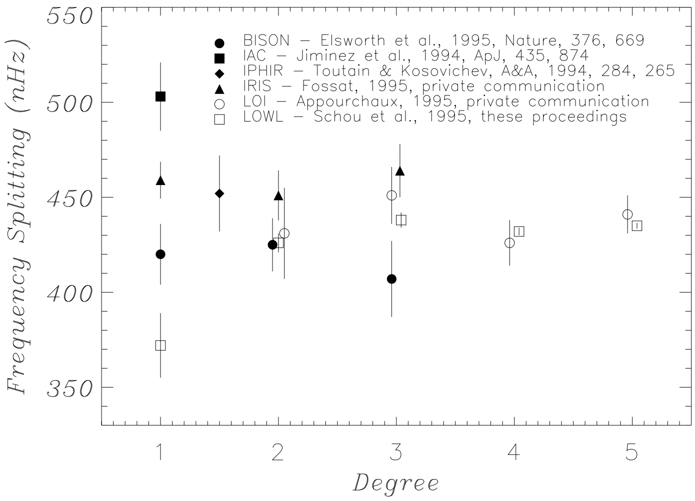

Fig. 1 Compilation of low-degree frequency splitting observations. Plot courtesy of T. Appourchaux.

that the errors associated with these observations have been severely underestimated. In this paper we shall argue in favor of the latter explanation.

2. Measurements

Early measurements of the low-degree frequency splittings tended to produce values clustering around 750 nHz but with some other measurements producing much lower values (see e.g. Harvey, 1988, for a review). An ℓ=1 frequency splitting of 750 nHz would imply a core rotation rate approximately an order of magnitude larger than the surface equatorial rate. Recent measurements, however, have tended to imply more moderate core rotation rates. A compilation of some recent measurements of the low-degree frequency splittings are shown in Figure 1. The plot indicates that the splittings are roughly consistent for mode degrees from 2 to 5, while the ℓ=1 observations are inconsistent within the quoted errors. The agreement for ℓ=2 is particularly encouraging, since these modes penetrate nearly as deeply as the ℓ=1 modes. A sharp change in the frequency splitting between ℓ=1 and ℓ=2 modes would imply a large gradient in the rotation rate with depth.

An important question, then, is whether the inconsistency of recent ℓ=1 measurements with one another and with past measurements is due to temporal variation in the core rotation rate or systematic errors in the measurements.

3. Systematic Errors

The nature and magnitude of systematic errors depend strongly on the particular instrument used and the data analysis employed. For this reason we will talk about systematic errors in general terms, but restrict specific comments to the instrument with which we are most familiar, the LOWL instrument.

The random errors quoted with observations are typically derived from the standard errors produced by the spectral fitting routines. These often underestimate the true random errors and increase the apparent discrepancy between various observations. These random errors must be viewed with caution.

Observed spectra of acoustic oscillations are complicated by the presence of temporal sidelobes if the duty cycle is less than 100%, and spatial leakage because only one solar hemisphere is observed. These spurious peaks can overlap with real peaks and cause systematic errors in the determination of frequencies and splittings.

To remove the effects of the temporal sidelobes, some observers have resorted to building observing networks. While the confusion of peaks caused by the temporal sidelobes has been well documented, very little work has been devoted to the nature and magnitude of the systematic errors these sidelobes produce. The combination of spatial and temporal leakage can cause the power peak of a given mode to coincide with the first temporal sidelobe of a mode differing in degree by one. This occurs when $\nu/L \approx 30$ μHz, where ν is the mode frequency and $L = \sqrt{\ell(\ell+1)}$. Thus, if the spectra are not fit properly in the analysis, a resonance type feature will be observed in plots of various mode parameters against ν/L at a value of ν/L of about 30 μHz (see e.g. Schou, 1992; Bachmann, Schou and Brown, 1993). Figure 2 shows a plot of the a_1 coefficient as derived from 1 year of data from the LOWL instrument as a function of ν/L (Schou et al., these proceedings). We see no significant resonance feature in these plots at 30 μHz to a level of a few nHz. The systematic error due to the interference between the ℓ=0 and 2 modes and their temporal sidelobes is significantly lower than this, since the spatial leakage between modes with $\Delta\ell$=2 is 2-3 times smaller than the leakage between modes with $\Delta\ell$=1. We thus conclude that temporal sidelobes do not have an important contribution to the systematic errors of the LOWL data.

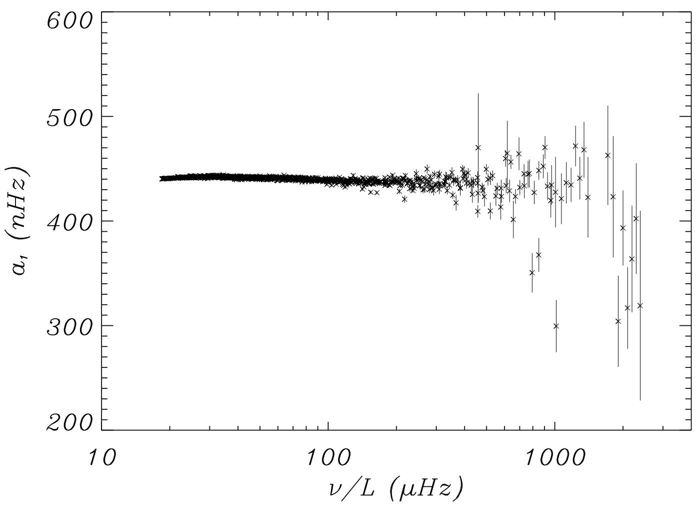

Fig. 2 The a_1 splitting coefficient as inferred from 1 year of LOWL data (see Schou et al., these proceedings).

However, both resolved and integrated light observations are hampered by the spatial leakage of modes within a given multiplet. In particular, the mixing of prograde and retrograde modes is probably the most serious source of systematic errors in low-degree frequency splitting measurements. This is especially the case for integrated light measurements where the observing geometry leads one to expect prograde and retrograde modes to be present with identical amplitudes and with a separation between $m = \pm 1$ modes in the same multiplet of order 1 μHz; a width which is comparable to the mode linewidth at 3 mHz. Several groups have recently confined their analysis to low-frequency modes which have reduced linewidths in order to minimize the effects of the mode blending. The observations presented by Fossat et al. (these proceedings) showing systematically different amplitudes for prograde and retrograde modes raises serious concerns about assumptions regarding the mode amplitudes. Even for spatially resolved observations, prograde and retrograde mode leakage is present in the complementary spectra, at a level which depends on the projection of the velocity vector and other geometrical factors. If these are not known precisely, then the leakage will be different from that anticipated, with resulting systematic errors.

In the case of the LOWL instrument, we observe the solar absorption line at only two points in opposite wings of the line. This instrument then has a velocity sensitivity which depends on the slope of the absorption line. The slope of the Potassium line which we use is in fact a function of position on the disk (Bonet et al., 1988) which introduces an apodization of sensitivity which we have not yet taken into account in our analysis. This causes an underestimation of the leakage and a corresponding underestimation of the frequency splitting. We find that this is most important for the ℓ=1 frequency splitting measurement and causes us to be suspicious of the very low values of the frequency splittings which we find for the ℓ=1 modes. To be cautious, we have performed rotation inversions not only of the entire LOWL 1 year dataset but also of a dataset excluding the ℓ=1 modes. These results are shown in Figures 3 and 4 and indicate a large difference in the inferred core rotation rate between the two cases. As expected, the rotation profile is unaffected above $0.2R_{\odot}$. Until we are able to rectify the analysis for the possible contamination of the ℓ=1 modes, we would conservatively adopt Figure 4 as our best estimate of solar internal rotation.

4. Discussion

We suggest that the wide range of disagreement of past low-degree frequency splitting measurements indicates that these measurements are subject to significant systematic errors. We would argue that early measurements which produced large values for the low-degree frequency splittings should be discarded in favor of the more recent measurements shown in Figure 1. There are several reasons why the current measurements are more reliable than the early measurements:

1) Longer time series are now available. Several groups are now analyzing time series of order one year which increases the dependability of the results for at least two reasons. The first is that the longer time series allows the determination of more accurate mode frequencies with random errors close to theoretical realization noise limits. The other

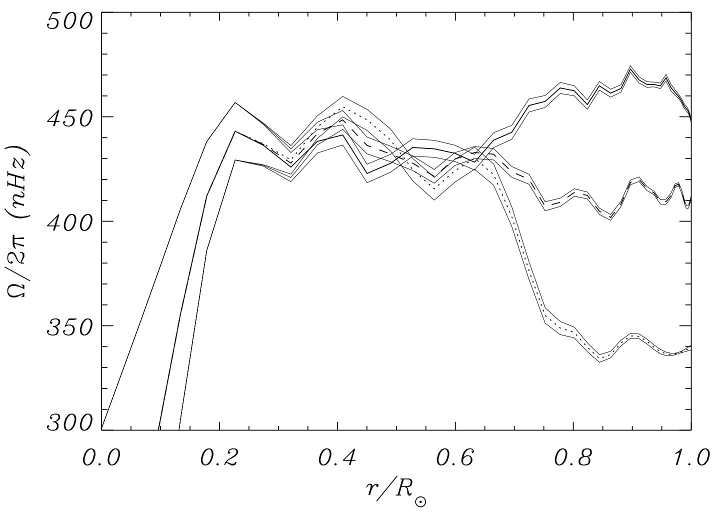

Fig. 3 Solar interior rotation profile as inferred from 1 year of LOWL data. Equatorial rotation is shown by the solid line, mid-latitude rotation by the dashed line, and polar rotation by the dotted line. The $\pm$ 1-σ errors are indicated by the thin solid lines.

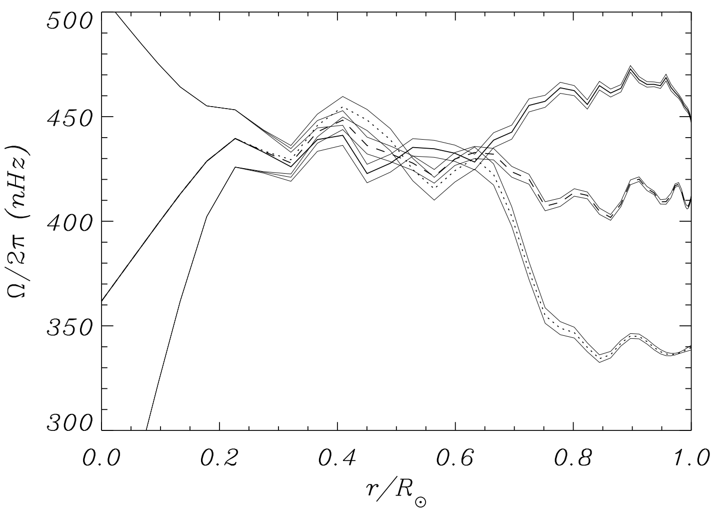

Fig. 4 Solar interior rotation profile as inferred from 1 year of LOWL data excluding the ℓ=1 modes. The line styles are the same as in Figure 3.

reason is that longer time series have allowed several groups to observe low signal-to-noise modes at low frequency where the linewidths are much narrower and therefore the systematic errors introduced by mode blending can be minimized.

2) Improved methods of mode parameter estimation. Analysis routines have become increasingly sophisticated and now generally include maximum likelihood fits which correctly model the statistics of the spectra and also include temporal and spatial leakage.

3) A wide variety of instruments now observe low-degree frequency splittings. These measurements include both spatially resolved and unresolved observations with a variety of technologies in both velocity and intensity. This number will soon be increased by the helioseismology instruments on the SOHO spacecraft. Hopefully, many independent observations will ultimately allow the identification and removal of systematic errors in low-degree frequency splitting measurements.

If the systematic errors associated with different measurements are uncorrelated then averaging the results from various studies can produce an average frequency splitting with reduced systematic error. Assuming that this is the case for the recent measurements shown in Figure 1, we calculate an unweighted average of these ℓ=1 splittings of 437 nHz with a scatter of 48 nHz. We would then argue that these observations, taken as a group, are consistent with low-degree frequency splittings which are independent of mode degree and imply a rigidly rotating solar core.

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