Helioseismic Constraints on the Gradient of Angular Velocity at the Base of the Solar Convection Zone

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ABSTRACT

The layer of transition from the nearly rigid rotation of the radiative interior to the latitudinal differential rotation of the convection zone plays a significant role in the internal dynamics of the Sun. Using rotational splitting coefficients of the p-mode frequencies, obtained during 1986-90 at the Big Bear Solar Observatory, we have found that the thickness of the transitional layer is 0.09$\pm$0.04 solar radii (63$\pm$28 Mm), and that most of the transition occurs beneath the adiabatically stratified part of the convection zone, as suggested by the dynamo theories of the 22-year solar activity cycle.

Subject headings: methods: data analysis — convection — Sun: rotation — Sun: interior — Sun: activity — Sun: oscillations

1. Introduction

Helioseismology has established the existence of a layer of strong gradients of angular velocity at the base of the solar convection zone (e.g. Brown *et al.*, 1989; Goode *et al.*, 1991; Tomczyk, *et al.*, 1995). This layer separates the convection zone exhibiting strong latitudinal differential rotation from the radiative interior rotating almost rigidly. Turbulence generated in the layer is likely to mix material in the upper radiative zone resulting in the observed deficit of Li and Be (e.g. Zahn, 1992). However, the theoretical estimates of the precise location and the thickness of the transition layer ('tachocline') depends on details of turbulent energy and momentum transport, and are uncertain (Spiegel & Zahn, 1992).

Perhaps, of the greatest interest, the transition layer is the most likely place for the solar dynamo (e.g. Weiss, 1994). Within this layer, the toroidal magnetic flux that appears at the surface in various forms of solar activity is generated from the radial component of the poloidal field (Branderburg, 1994). The toroidal flux is believed to be mainly accumulated in a thin layer just beneath the convection zone because convection would quickly destroy the toroidal flux if the layer were widely extended into the convection zone. However, as recently argued by Rüdiger and Brandenburg (1995), this layer cannot be very thin because the period of the solar cycle, which depends on the turbulent magnetic diffusion time through the layer, would be too short. They estimated the thickness to be at least $0.05R \approx 35$ Mm $\approx \frac{1}{2}H_p$, where R is the solar radius and H_p is the local pressure scale height.

Estimates of the thickness and precise location of the transition layer by standard helioseismic inversion techniques from rotational splitting of oscillation p modes are rather uncertain. Attempts to resolve the layer under global smoothness constraints lead to either an oversmoothed angular velocity profile or to spurious oscillations around the transition layer (cf Goode *et al.*, 1991). Thompson (1990) has demonstrated that even if the transition were discontinuous the formal helioseismic inversions still produce a broad smooth region. To overcome these difficulties, Goode *et al.* (1991) assumed that the transition layer is, in fact, discontinuous, and found that their model fits the helioseismic data best when the discontinuity coincides with the base of the convection zone. In this Letter, we demonstrate that the discontinuous model of solar rotation is not the best fit to the data, and that a model with a transitional layer of finite thickness ($\approx 0.09 \pm 0.04R$) fits the data more accurately than the discontinuous model. The midpoint of this layer is found at $0.692 \pm 0.005R$, slightly below the the convection zone.

2. Frequency Splitting and Solar Rotation Law

Observed p-mode rotational frequency splitting is traditionally represented in the form (Duvall, *et al.*, 1986)

$$\Delta\nu_{nlm} = L \sum_{k=0,1,2,\ldots} a_{2k+1}(n,l) P_{2k+1}\left(\frac{m}{L}\right), \quad (1)$$

where n, l, and m are the radial order, the angular degree, and the angular order of a normal mode respectively; $L = \sqrt{l(l+1)}$; P_k are Legendre polynomials. Thus, the observational data are sets of the 'odd a-coefficients', a_{2k+1}, for mode multiplets (n,l).

For the modes of intermediate and high degree, l, which probe the convection zone, the a-coefficients are directly related to the radial function $A_k(r)$ of the solar rotation law represented in terms of associated Legendre functions of order 1, $P_k^1(\theta)$ (Kosovichev, 1988) and expressed as

$$\Omega(r,\theta)/2\pi = \sum_{k=0,1,2,\ldots} \alpha_k A_{2k+1}(r) \frac{P_{2k+1}^1(\theta)}{\sin\theta}, \quad (2)$$

where $\alpha_k = (-1)^k \frac{k!2^k}{(2k+1)!!}$. The relation between $a_k(n,l)$ and $A_k(r)$ is

$$a_k(n,l) = \frac{1}{I_{nl}} \int_0^R A_k(r) \left[U_{nl}^2 + l(l+1)V_{nl}^2 - 2U_{nl}V_{nl} - V_{nl}^2\right] \rho r^2 dr, \quad (3)$$

where U_{nl} and V_{nl} are the radial and horizontal displacement eigenfunctions of oscillation modes, $\rho(r)$ is the density, and $I_{nl} = \int_0^R \left[U_{nl}^2 + l(l+1)V_{nl}^2\right] \rho r^2 dr$ is the mode inertia. The contribution of the last two terms in the brackets of equation (3), resulting from the Coriolis effect, does not exceed a few percent, and, therefore, can be treated as a small perturbation. Then, we introduce coefficients a' as

$$a_k(n,l) \equiv a'_k(n,l)\left(1 - C_{nl}\right), \quad (4)$$

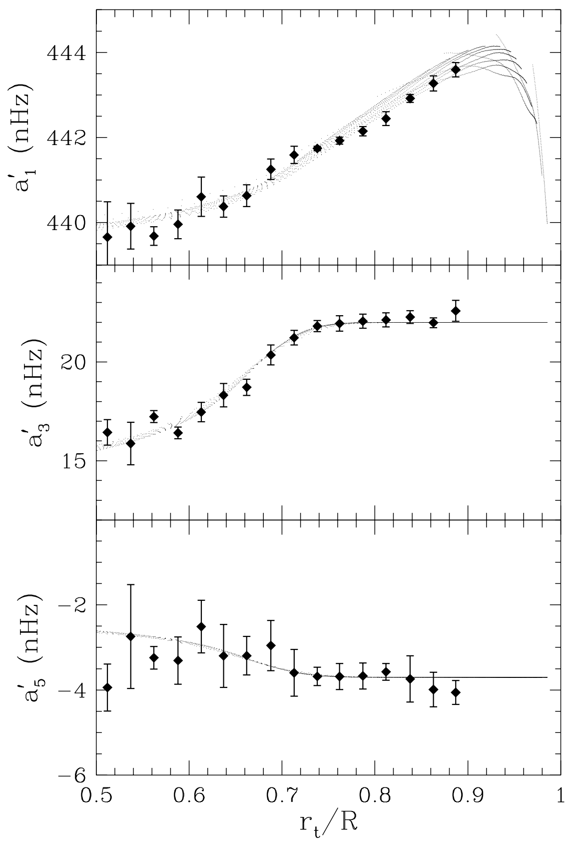

Fig. 1.— Seismic averages (eq. [5]) of the radial function, $A_k(r)$, of the rotation law (eq. [2]), as functions of the radius of the mode lower turning point, r_t. The dots represent the theoretical values computed for approximately 3,000 modes of $l = 5 - 250$ using the model of solar rotation, shown in Figure 2. The diamonds with the error bars show the averaged $a'_k(n,l)$ obtained from the BBSO 1986-90 data ($l = 5 - 60$).

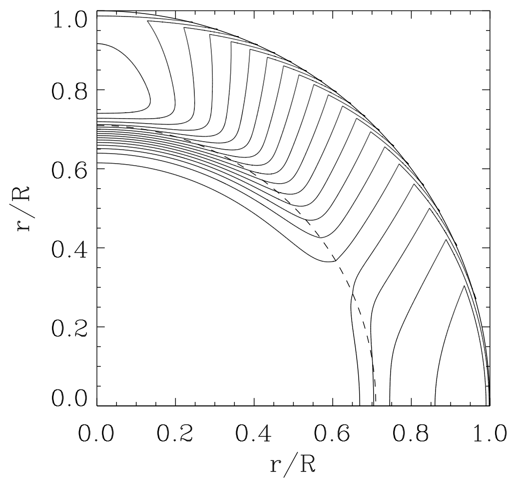

Fig. 2.— Contours of $\Omega(r,\theta)/2\pi$ from 328 nHz to 461 nHz separated by 7 nHz of a solar rotation model used in Figure 1. The dashed curve shows the lower boundary of the convection zone. In this model, based on the first three terms of equation (2), $A_1(r)$ is approximated, below the convection zone, by a constant, and, in the convection zone, by two linear functions, the first of which was chosen to fit gradually increasing a'_1, while the second function (describing the subsurface shear layer) was found by matching the internal rotation with the surface rotation (Snodgrass, 1992) assuming the conservation of the angular momentum in the subsurface layer (Foukal and Jokipii, 1975). The estimated thickness of the subsurface layer is approximately 12 Mm ($0.017R$). Functions $A_3(r)$ and $A_5(r)$ are taken in the form of equation (6).

where

$$a'_k(n,l) \approx \frac{1}{I_{nl}} \int_0^R A_k(r) \left[U_{nl}^2 + l(l+1)V_{nl}^2\right] \rho r^2 dr. \quad (5)$$

New coefficients $a'_k(n,l)$ represent the 'seismic averages' of the radial function A_k with the weighting function proportional to the mode energy density, and C_{nl} is a constant which describes the effect of the Coriolis force (Ledoux, 1951).

For high-frequency p modes, $a'_k(n,l)$ are essentially functions of only one asymptotic variable: the angular phase speed ω_{nl}/L, or equivalently, the radius of the lower turning point r_t of the modes (Gough, 1984). This asymptotic behavior is demonstrated in Figure 1, in which small dots show the seismic averages for a model of solar rotation depicted in Figure 2. We use this asymptotic property of the splitting coefficient for the analyses of the observational data by averaging $a'_k(n,l)$ over short intervals of r_t.

3. Estimates of Width and Location of the Transition Layer

We have used the splitting a-coefficients (a_1, a_3, and a_5) of approximately 800 multiplets of $l = 5 - 60$ obtained by Woodard and Libbrecht (1993) from 1986 and 1988-90 observations made at the Big Bear Solar Observatory (BBSO). The a-coefficients corrected for the Coriolis effect, $a'_k(n,l)$, for each year have been grouped and weighted average computed in 16 equal intervals of r_t, between $0.5R$ and $0.9R$, each being $0.025R$ wide. Then, the weighted averages for the four years have been computed. The final averages, $< a'_{k,j} >_{\rm obs}$ ($j = 1, ..., 16$), are shown by the diamonds with the error bars in Figure 1. The variation at the base of the convection zone is particularly strong for a'_3. Therefore, we have used this coefficient to estimate the thickness and the location of the transition layer. The absence at the base of the convection zone of a sharp variation of a'_1, which represents the seismic average of the net angular momentum of a spherical shell, means essentially zero net torque between the radiative and convection zones (Gough, 1985; Gilman *et al*, 1989). We shall discuss the limits of the variation of a'_1 in a future publication.

Helioseismic inversions have revealed that the radiative interior essentially rotates rigidly. Therefore, in rotation law (2), functions $A_k(r)$ for $k > 1$ are approximately equal to zero in the radiative zone. Also, the averaged $a'_3(n, l)$-coefficients shown in Figure 1 suggest that A_3 is almost constant in the convection zone. Therefore, we have considered A_3 in the parametric form

$$A_3(r) = A_{3,0}\Phi(r), \qquad (6)$$

where $\Phi(r) = 0.5\,(1 + {\rm erf}\,[2(r - r_0)/w])$, erf is the error function, r_0 is the radius of the central point of the transition zone, and w is the characteristic thickness corresponding to the variation of $\Phi(r)$ from 0.08 to 0.92. The value of $A_{3,0}$ (≈ 22 nHz) is determined from the flat part of a'_3 in the convection zone. Then, we have computed $< a'_3(n, l) >_{\rm mod}$ coefficients for equation (6) using equation (5) and the same averaging procedure that was applied to the BBSO data, and, finally, determined the mean squared difference

$$\chi^2(w, r_0) = \sum_{j=1}^{16} \frac{1}{\sigma_j^2} \left[< a'_{3,j} >_{\rm obs} - < a'_{3,j} >_{\rm mod}\right]^2 . \qquad (7)$$

Function $\chi^2(w, r_0)$ shown in Figure 3 reaches a minimum of 16.8 at $r_0 \approx 0.692R$ and $w \approx 0.09R$. The boundaries of the shaded area correspond to the increase of χ^2 by 1, or to 1 standard deviation in r_0 and w. The similar result was obtained using different approximations for the transition function, $\Phi(r)$, and, also, choosing different intervals of r_t for averaging the data. The analysis repeated with an extended BBSO data set ($l = 5 - 140$) provided to us by P.R. Goode gave the same result.

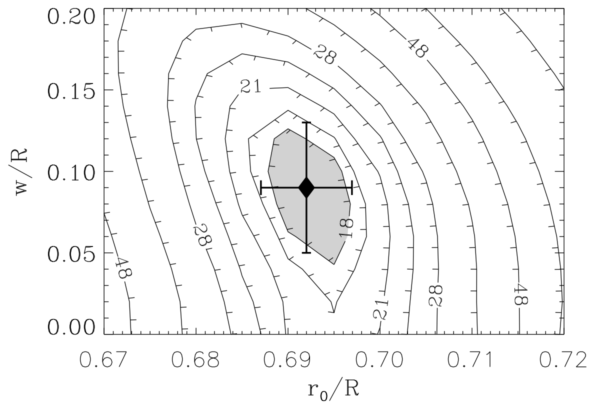

Fig. 3.— Contours of $\chi^2(w, r_0)$ evaluated from equation (7) at $\chi^2 = 18, 19, 21, 24, 28, 36, 48, 64$ and 96; w is the thickness of the transition layer, r_0 is its central radius. The shaded area corresponds to the increase of χ^2 by 1 from its minimum value, or 1σ uncertainty in the parameters. The error bars show the 1σ uncertainty estimated from statistical modeling by adding Gaussian noise to the data.

We note that the position of the center of the transition layer is determined more accurately from the data than is the thickness. It is evident the center of the layer is beneath the boundary of adiabatic stratification of the convection zone which is at $0.713 \pm 0.003R$, and coincides with the zone of the sharp variation of the sound speed (Figure 4). For Goode-Dziembowski's (1991) model with a discontinuity at the base of the convection zone ($w = 0$, $r_0 = 0.71R$) $\chi^2 \approx 36$, is significantly higher than the minimal value. Therefore, their model with a very thin transition layer at the base of the convection zone can be excluded.

4. Conclusion

Estimated from the 1986-90 BBSO solar oscillation data, the thickness ($0.09 \pm 0.04R$) and the central position ($0.692 \pm 0.005R$) of the transition layer from the nearly uniform rotation in the radiative interior to the differential rotation in the convection zone at the base of the solar convection zone are generally consistent with the requirements of the $\alpha\omega$-dynamo theory of the 22-year solar activity cycle (Rüdiger and Brandenburg, 1995). Our results show that most of the transition occurs beneath the zone of adiabatic convection. However, the upper boundary of the transi-

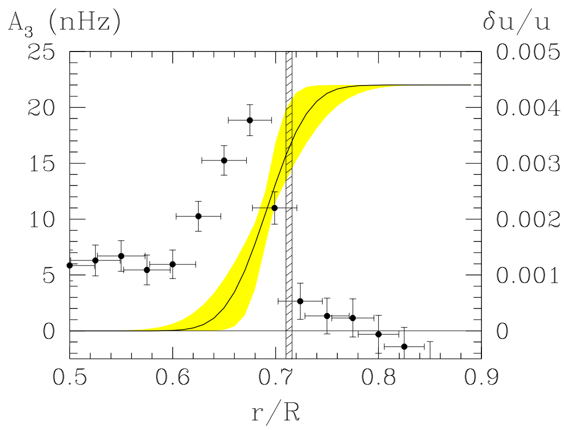

Fig. 4.— The solid curve with the shadow (indicating 1σ uncertainty) shows the parameter A_3 of rotation law (2) estimated from the BBSO data. The points with error bars represent the variations of the ratio of the pressure to the density, $u \equiv p/\rho$, relative to a standard solar model, inferred from the GONG data (Gough *et al*, 1996). The vertical hatched column shows the location of the base of the adiabatically stratified part of the convection zone, as determined by Christensen-Dalsgaard *et al.* (1991) and by Kosovichev and Fedorova (1991).

tion layer extends into the convection zone. To determine whether the transition layer coincides with the zone of convective overshoot is an important problem of helioseismology.

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