Inferences of Element Abundances from Helioseismic Data

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Abstract. The abundance of helium in the Sun's interior is estimated by the method of 'model-oriented' helioseismic inversion of solar oscillation frequencies. In this method, the equations of state and thermal balance are used in addition to the hydrostatic equation. By inverting the observed p-mode frequencies, direct evidence for gravitational settling of helium has been obtained. However, the helium settling cannot account for the anomalously low helium abundance of the solar corona and wind. The ratio of the helium to hydrogen densities in the convection zone inferred from the helioseismic data [1,2] using the most recent equation of state [3] is $8.5\% \pm 0.2\%$ suggesting that the separation of helium and hydrogen predominantly occurs in the solar atmosphere and corona.

INTRODUCTION

Helioseismic inversion of solar oscillation frequencies is a unique tool for inferring the physical properties of the Sun's interior. The frequencies of several thousand acoustic modes of the oscillations in the 5-min range have been measured to a very high precision, with typical uncertainty limited to several parts in a million. The SOHO space mission [4] and the GONG network [5], which are currently in progress, will provide even more precise frequencies of the p modes. Helioseismic data are commonly used to infer the stratifications of the density and the sound speed by inversion(e.g. [6]).

Helioseismology can also provide information about the chemical composition of the interior of the Sun. This information can be obtained from two basic methods. The first, indirect, method calibrates theoretical solar models by comparing either the observed oscillation frequencies with the eigenfrequencies of the models, or the primary seismic parameters (e.g. the sound speed, c^2, the density, ρ, and the adiabatic exponent, γ) inferred from the observed frequencies with the corresponding parameters of the solar models. The second approach measures element abundances by direct, 'model-oriented', inversion of the frequencies, incorporating additional equations of the stellar structure into the helioseismic inverse problem. The additional equation to estimate the composition of the convection zone is the equation of state that relates variations of the adiabatic exponent in the zones of ionization of the elements to their abundances. In the radiative interior, where the most abundant elements are almost totally ionized, the energy equations together with equations of the energy generation rate and the opacity are used to relate primary seismic parameters with the abundances.

THE HELIOSEISMIC INVERSION METHOD

For frequency inversions, the linearized integral equations relating frequency perturbations to variations of the solar structure are derived from a variational principle. These equations are transformed to depend on a chosen pair of deviation variables $\boldsymbol{f}$ (e.g. $\boldsymbol{f}= (\delta \ln \rho, \delta \ln \gamma)$ that are assumed to be functions of the radius, r, alone, yielding (e.g. [6])

$$\frac{\delta \nu_i}{\nu_i} = \int_0^{\mathrm{R}_\odot} \boldsymbol{K}^{(i)}_{\boldsymbol{f}} \cdot \boldsymbol{f} \, \mathrm{d}r + \frac{F(\nu_i)}{E_l(\nu_i)}, \quad i = 1, ..., N. \tag{1}$$

Here $\delta\nu_i$ is the frequency difference between the eigenfrequency ν_i of a solar model and the corresponding frequency of the Sun; $\boldsymbol{K}^{(i)}_{\boldsymbol{f}}(r)$ is the seismic integral kernel for structure variable, $\boldsymbol{f}$, which depends on the mode eigenfunction; $\mathrm{R}_\odot$ is the radius of the Sun; and $E_l(\nu_i)$ is the mode inertia. The arbitrary function $F(\nu_i)$ is added to take into account surface effects. The subscript i labels the modes; N is the total number of modes in a data set.

The structure variable $\boldsymbol{f}$ can be of two types: 'primary', e.g. the density, ρ, and the adiabatic exponent ,γ, or 'secondary' (model-oriented), e.g. the abundances of helium, Y, and heavy elements, Z, where Y is the fractional mass of helium, and Z is the fractional mass of the elements heavier than helium. The abundance of hydrogen X equals $1 - Y - Z$. For the 'primary' structural variables, pressure p, ρ and γ, and their combinations, the helioseismic equations (1) are derived under the basic assumptions about the solar structure: spherical symmetry and hydrostatic equilibrium; whereas for the 'secondary' parameters additional structure equations must be considered. Only two structure variables appear in equation (1) because the equations of hydrostatic support

$$\frac{dp}{dr} = -\frac{Gm\rho}{r^2}, \qquad \frac{dm}{dr} = 4\pi\rho r^2 \tag{2}$$

are used.

Two main options for determining the 'secondary' variables have been studied. The first option uses the equation of state in the form

$$\gamma = \gamma(p, \rho, Y, Z). \tag{3}$$

Since the variations of the adiabatic exponent, γ, occur mainly in the zones of ionization of helium and hydrogen at the top of the convection zone, the frequency variations can be expressed in terms of the uniform helium abundance in the convection zone (assuming that the abundance of heavier elements, Z, is known) and one of the hydrostatic variable, e.g. density or the ratio $u = p/\rho$.

The second option considers a full set of structure equations, including the equations of thermal balance and energy transport, thus introducing 'non-seismic' variables, such as temperature, element abundances and opacities in the radiative zone, into the seismic equations.

Equation (1) can be transformed into a relation between $\frac{\delta\nu_i}{\nu_i}$ and composition deviations δY and δZ by additionally imposing the constraint of thermal balance through the equations

$$\frac{dL}{dr} = 4\pi\rho r^2\epsilon, \tag{4}$$

$$\frac{dT}{dr} = \begin{cases} -\dfrac{3\kappa\rho L}{64\pi\sigma r^2 T^3} & \text{in radiative zones} \\ \left(\dfrac{dT}{dr}\right)_c & \text{in the convection zone} \end{cases} \tag{5}$$

where L is luminosity, T is temperature, ϵ is the energy-generation rate per unit mass, κ is opacity and σ is the Stefan-Boltzmann constant. The function $(dT/dr)_c$ is provided by the mixing-length formalism relating the temperature gradient to energy transport in the convection zone. In addition, the functions of $\epsilon(\rho, T, Y, Z)$ and $\kappa(\rho, T, Y, Z)$ and their partial derivatives are required. The transformation then yelds new perturbation relations which may be written

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{\mathrm{R}_\odot} \boldsymbol{K}_{\boldsymbol{g}}^{(i)} \cdot \boldsymbol{g}\,\mathrm{d}r + \frac{F(\nu_i)}{E_l(\nu_i)}, \quad i = 1, ..., N \tag{6}$$

where $\boldsymbol{g} = (\delta Y, \delta Z)$.

Computing thes seismic kernels in equation (6) is straightforward. The linearized structure equations (2)-(5) can be expressed formally

$$\mathcal{B}\boldsymbol{f} = \boldsymbol{g}, \tag{7}$$

where $\mathcal{B}$ is a linear differential operator. This equation must be supplemented with appropriate boundary conditions, that are derived from the requirement that conditions in the photosphere are unchanged, which we adopt in the form $\ln\rho = 0$, $\ln L = 0$ at $r = R$. Substituting equation (7) into equation (6) yields

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{\mathrm{R}_\odot} \boldsymbol{K}_{\boldsymbol{g}}^{(i)} \cdot \mathcal{B}\boldsymbol{f}\,\mathrm{d}r + \frac{F(\nu_i)}{E_l(\nu_i)} = \int_0^{\mathrm{R}_\odot} \mathcal{B}^*\boldsymbol{K}_{\boldsymbol{g}}^{(i)} \cdot \boldsymbol{f}\,\mathrm{d}r + \frac{F(\nu_i)}{E_l(\nu_i)}, \tag{8}$$

where $\mathcal{B}^*$ is the adjoint of $\mathcal{B}$. By demanding that the expressions on the right-sides of equations (1) and (8) be identical, we obtain

$$\mathcal{B}^* \boldsymbol{K}_{\boldsymbol{g}}^{(i)} = \boldsymbol{K}_{\boldsymbol{f}}^{(i)}. \tag{9}$$

Thus, the seismic kernels for the element abundances are obtained as solutions of the adjoint linearized structure equations.

The procedure of solving the seismic equations (1) or (6) for a set of observed modes basically consists of constructing linear combinations of the equations, which provide localized averages of the structure parameters in the form

$$\overline{\boldsymbol{f}}(r_0) = \int_0^{\mathrm{R}_\odot} A_{\boldsymbol{f}}(r_0, r) \boldsymbol{f} \, \mathrm{d}r. \tag{10}$$

We have applied the Backus-Gilbert optimally localized averaging technique [7]. Consider, for instance, inversion for the abundances of helium, $Y(r)$, and heavy elements, $Z(r)$. In this case, $\boldsymbol{f} = (\delta Y, \delta Z)$, and helioseismic equations take the form:

$$\frac{\delta \nu_i}{\nu_i} = \int_0^R K_{Y,Z}^{(i)} \, \delta Y \, \mathrm{d}r + \int_0^R K_{Z,Y}^{(i)} \, \delta Z \, \mathrm{d}r + \frac{F(\nu_i)}{E_l(\nu_i)}. \tag{11}$$

The Backus-Gilbert method is to construct appropriate linear combinations

$$\sum_i c_i(r_0) \frac{\delta \nu_i}{\nu_i} \tag{12}$$

of the data from coefficients $c_i(r_0)$ designed to make unimodular averaging kernels for one of the structure variables, say, Y:

$$A_{Y,Z}(r_0; r) = \sum_i c_i(r_0) K_{Y,Z}^{(i)}(r) \tag{13}$$

well localized about a chosen location $r = r_0$ and the corresponding averaging seismic kernels for the second variable, Z, $A_{Z,Y}(r_0; r) = \sum_i c_i(r_0) K_{Z,Y}^{(i)}(r)$ and the 'surface' term $\sum c_i(r_0) F(\nu_i)/E_l(\nu_i)$ small everywhere. Then, the linear combination (12) represents an average $\overline{\delta Y}$ of δY, given by

$$\overline{\delta Y}(r_0) \equiv \int_0^R A_{Y,Z}(r_0, r) \, \delta Y(r) \, \mathrm{d}r \approx \sum_i c_i \frac{\delta \nu_i}{\nu_i} \tag{14}$$

localized in the vicinity of $r = r_0$. If both the equation of state (3) and the equations of thermal balance (4)-(5) are imposed in addition to the hydrostatic equations (2) then the localized averages of the helium abundance, $\overline{\delta Y}(r_0)$, can be obtained in most of the solar interior (Fig. 1a). With the equation of state alone, the localized averages can be obtained only in the helium and hydrogen ionization zones, just beneath the solar surface (Fig. 1b).

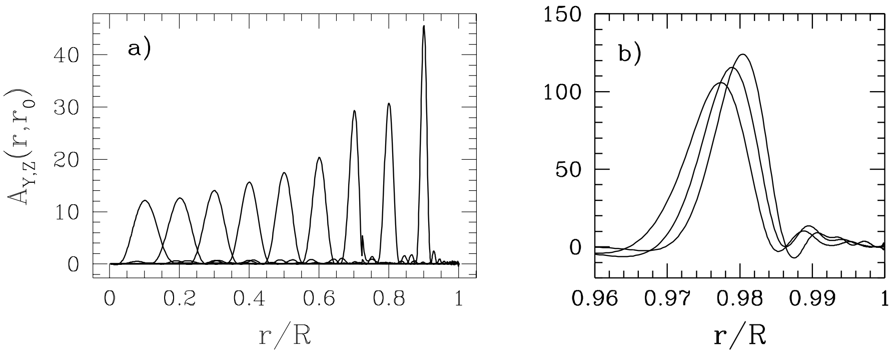

FIGURE 1. A selection of the localized averaging kernels for the helium abundance obtained using (a) both the equation of state (3) and the thermal balance equations (4)-(5), and (b) only the equation of state, as the additional constraints. The averaging kernels were obtained as linear combinations of the seismic kernels, $K^{(i)}_{Y,Z}$, of about 600 p-modes of oscillations in the 5-min range. The typical radial resolution near the solar center is $0.05 - 0.07R$. The maximum of the averaging kernel closest to the center, that can be constructed for the p modes, is about at $0.05R$.

THE INVERSION RESULTS

The helioseismic data used in this analysis are combinations of 16 frequencies of low-degree modes ($l = 0$, 1 and 2) in the frequency range $2.5 \lesssim \nu \lesssim 3$ mHz taken from the IPHIR data set [1] and 598 frequencies of intermediate-degree modes ($l = 4$–140, $\nu = 1.5$–3 mHz) observed at BBSO in 1988 [2].

We have carried out the inversion for the helium abundance using both types of the additional constraints discussed in the previous sections: the equation of state together with the thermal balance equations and the equation of state alone. For the reference, we have adopted a standard solar model [8] obtained under the assumption that in the course of solar evolution the distribution of the element abundances is changed only as a result of nuclear fusion in the central core. It is assumed that mixing takes place only in the convection zone where the element abundances are the same as they were at the beginning of the solar evolution on the Main Sequence about 4.75×10^9 years ago. In the convection zone of this model, the abundance of helium $Y_0 \approx 0.28$ (n[He]/n[H]$\approx$ 10%), and the abundance of heavy elements Z_0=0.02.

Figure 2 shows the optimal averages for the difference of Y between the Sun and the model inferred by inverting the solar frequencies imposing both the equation-of-state and the thermal balance constraints. We have used the tables of the thermodynamic properties and opacity of the solar plasma from OPAL [3], and the nuclear reaction rates by Caughlan and Fowler [9]. From Figure 2, it is evident that the helium abundance in the convection zone is approximately 2.5% lower than in the reference model. The most probable explanation for the helium depletion is gravitational settling. A similar con-

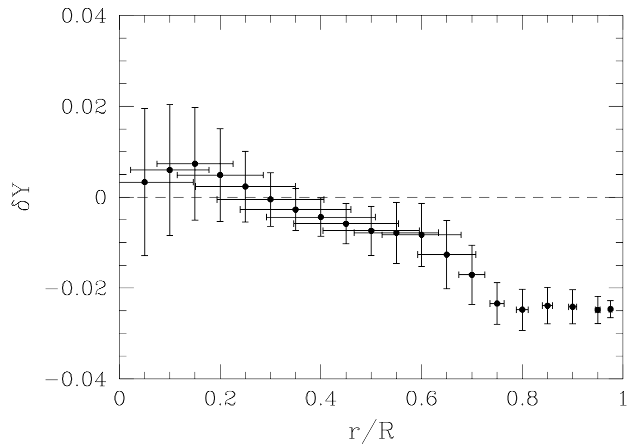

FIGURE 2. Optimally localized averages of the difference δY, where Y is the abundance of helium, between the Sun and the reference solar model [8], inferred from combinations of intermediate-degree mode frequencies from BBSO [2] and of the low-degree mode frequencies from IPHIR [1]. The horizontal bars represent the resolution lengths; the vertical bars represent standard errors. Both the equation-of-state and the thermal balance constraints are imposed.

clusion based on the results of the 'primary inversion' for the sound speed was made by Christensen-Dalsgaard *et al.* [10]. Gravitational settling of helium and heavier elements is often neglected in the stellar evolution theory because of the belief that mixing resulting from meridional circulation largely compensates the settling [11]. The variations of δY in the radiative zone ($r/R < 0.7$) most likely also result from helium diffusion towards the center. The diffusion is more efficient in the upper part of the radiative zone ($0.6 < r/R < 0.7$) than in the deep interior. That explains the relatively steep variation of δY in the layer just beneath the convection zone. The distribution of elements in this layer is likely to also be affected by rotationally induced mixing [12]. This mixing could explain the deficit of lithium and beryllium observed on the solar surface if mixing beneath the convection zone reaches the depths where the elements are destroyed in nuclear reactions. However, the details of this process are unknown, and helioseismology has provided only indirect evidence for the mixing beneath the convection zone [13]. Helium is slightly underabundant in the solar core resulting in a slight increase of the theoretical solar neutrino fluxes [14].

The helium abundance in the convection zone can be obtained using the equation of state as the only constraint in addition to the hydrostatic equations. In this case, the helium abundance is estimated from the variation of γ in the HeII ionization zone. In principle, this determination should be less uncertain than the previous estimate which, in addition, depends on the accuracy of the thermal balance constraint. However, the inversion results are found to be very sensitive to the details of the equation of state (Fig. 3). A

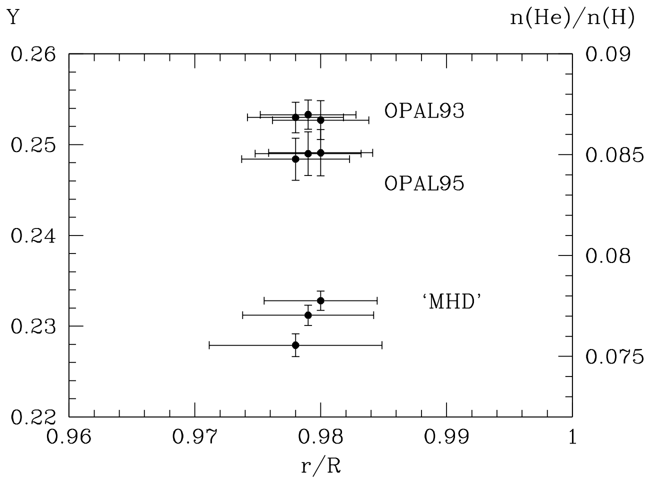

FIGURE 3. Optimally localized averages of the helium abundance Y in the convection zone, inferred from the combination of BBSO [2] and IPHIR [1] data using two different models of the equation of state: OPAL [3] and 'MHD' [12].

version of the so-called 'MHD' equation of state [15] gives the helium abundance $Y \approx 0.232 \pm 0.006$, whereas the OPAL equation of state [3] leads to $Y \approx 0.254 \pm 0.006$. The most recent version of the OPAL equation of state gives a slightly lower value: 0.248 ± 0.006. The difference between the MHD and OPAL equations of state has not been fully resolved. However, both estimates are substantially lower than the standard model value, $Y = 0.280$, and therefore, are consistent with the general picture of helium settling.

DISCUSSION

The abundance of helium in the solar interior is inferred from the observed frequencies of solar p modes by inversion using the equation of state and the equation of thermal balance as additional constraints. The accuracy of these inversions depends not only on the accuracy of the helioseismic data, but also on the correctness of the additional constraints based on theoretical models of the thermodynamic and radiative properties of the solar plasma. The uncertainties in the microscopic properties may result in systematic errors in the helioseismic measurements of the elemental abundances. Using the most complete models of the equation of state and opacity from OPAL [3], we have determined the distribution of the helium abundance in the interior, and found direct evidence for gravitational settling of helium in the course of the solar evolution. The helium abundance, Y, in the convection zone inferred using the OPAL equation of state is approximately 0.25 (n[He]/n[H]$\approx$ 8.5%). Using a version of the 'MHD' equation of state we have found $Y \approx 0.23$ (n[He]/n[H] $\approx$ 7.7%). It is not established yet why the OPAL and 'MHD' models of solar thermodynamics give such different results. However, with both models, the helium abundance inferred from helioseismic data [1,2] in the convection zone is substantially higher than the helium abundance in the solar corona and wind [16], suggesting that a separation of elements occurs not only in the interior but also in the solar atmosphere.

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