

THE UPPER CONVECTIVE BOUNDARY LAYER

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ABSTRACT

I review the progress that has been made in our understanding of the physics of the upper boundary layer of the Sun, its influence on frequencies of five-minute oscillations, and its role in excitation of the oscillations. I also discuss approaches to seismological diagnosis of the properties of the layer, important information about which will be obtained from MDI high-resolution data.

Keywords: *Sun's interior, oscillations, convection, turbulence, Reynolds stresses, nonadiabatic effects.*

1. INTRODUCTION

In most of the convection zone the temperature gradient is nearly adiabatic because of the high efficiency of energy transfer by low-speed convective motions. However, the gradient becomes superadiabatic and convective velocities substantially increase near the surface where the convective transfer is inefficient owing to low density. The superadiabatic boundary layer, which occupies only a few hundred kilometres below the surface, is the region of the greatest helioseismological uncertainty. The assumptions about hydrostatic equilibrium of the solar structure and adiabaticity of oscillations that are commonly used in helioseismic inversions are not valid in this layer. This uncertainty may affect our conclusions about properties of the deep interior. Our potential for improving the accuracy of inversions is quite limited unless the physics of interaction between convection and oscillation modes and also the structure of the layer are understood better.

The upper convective layer is likely to be the place where p modes are excited. Information about the excitation and damping is contained in the amplitudes and phases of the oscillations, and also in the shapes of spectral lines in the power spectra. Despite considerable efforts to understand the mechanisms of excitation and damping of the modes, the current state of the art is still far from completion and is not in full agreement with observations.

Helioseismology offers the opportunity to study subsurface flows, and therefore to test the numerical predictions and to improve the theoretical models of turbulent convection. Several techniques are available to measure subphotospheric flows. They employ local seismic analysis,

based on measuring some local properties of wave propagation, such as the dispersion relation or travel time, and are reviewed by F. Hill in these proceedings. The inhomogeneous structure of the surface layer results in complicated splitting and shifting of the oscillation eigenfrequencies, from which one could possibly infer characteristics of the structures (e.g. Lavelly & Ritzwoller, 1992). However, the analysis requires accurate measurements of frequencies of individual modes, which are expected from SOHO and GONG.

Presently, most of the helioseismic studies are concentrated on the mean spherically symmetrical structure of the Sun's interior. These studies also provide valuable information for testing theories of convection. The theoretical background of the studies is provided by the so-called diagonal sum rule (Gilbert, 1971) which states that the average frequency of a split multiplet is the degenerate eigenfrequency of the spherically symmetric component of the solar structure, provided that the aspherical component can be considered as a small perturbation. This assumption is almost certainly not justified in the convective boundary layer. However, it is commonly used because a general analysis of non-linear aspherical perturbations is very complicated (cf Kosovichev & Perdang, 1988).

2. STRUCTURE OF THE CONVECTIVE BOUNDARY LAYER

2.1. Observations

High-resolution observations of the Sun, such as those obtained with the Solar Optical Universal Polarimeter (SOUP) on Spacelab 2 (Title *et al.*, 1989), reveal the highly inhomogeneous structure of the solar surface covered by convective granules. The granules display significant variations of temperature and density, accompanied by strong flows, the velocity of which may reach the local sound speed. The highest velocities are observed in convective downflows that form thin dark lanes around relatively bright areas of ascending flow. The typical size of granules is about 10^3 km. Granules form two patterns of larger scale on the surface: mesogranulation ($\approx 5 \times 10^3$ km) ultimately related to large "exploding" granules, and

supergranulation that has a scale of about 3×10^4 km and is probably tied to deep convective flows.

2.2. Numerical simulations

The observed horizontal structure of the surface convection is reproduced remarkably well in 3-D numerical simulations (e.g. Cattaneo, *et al.*, 1991; Rast, *et al.*, 1993). Quantitative agreement has been achieved between the simulations and ‘surface’ observables, such as spectral line widths and intensities (Spruit, *et al.*, 1990; Atroshchenko & Gadun, 1994).

The simulations also provide an important insight into the vertical structure of the convective boundary layer. They have demonstrated that convection consists of two primary components of flow: relatively slow broad convective updraft, relatively smooth and weak in turbulence, and fast narrow downdrafting plumes. The boundaries between the downflowing plumes and the surrounding upflows are regions of strong shear, and therefore, of strong turbulence. The “exploding” granules generate particularly sharp downdrafting plumes at their centres (Rast, 1995). Descent velocities in such plumes can become supersonic, and therefore the plumes can be sites of significant acoustic excitation. Numerical simulations also show organization of subsurface flows into large-scale structures which have not yet been understood. Helioseismology gives an opportunity to probe directly the structure and flows beneath the surface and, thus, to test numerical models of convection.

2.3. Mean stratification

While there have been attempts to deduce a model of the spherical structure by averaging the results of the numerical simulations (e.g. Nordlund & Dravins, 1990; Atroshchenko and Gadun, 1994), the standard description of the mean stratification of the convective layers is based on mixing-length models. In the mixing-length approach (e.g. Böhm-Vitense, 1958; Cox & Giuli, 1968), the complicated situation in the actual convection zone is replaced by a group of identical convective elements, “bubbles”, all of which have the same properties at a given distance, r , from the solar centre. These convective elements are not drastically different from the surrounding. They rise or fall under the buoyancy force through a characteristic distance, l , the mixing length, and then break up and mix with the surroundings. At any radius r , the convective elements are assumed all to have the same speed, V_{conv} , which approximates the average speed of the actual convective elements.

The structure of the bulk of the convection zone is very close to being isentropic because the plasma density is high enough that convective flows of low speed are sufficient to transport the convective energy. However, near

the surface, even at high convective velocities, convection can less easily transfer the heat, owing to the low density. The stratification of the zone of ineffectual convection, which is only about 500 km thick, is substantially superadiabatic, resulting in significant buoyancy force and rapid acceleration of the convective elements. Nevertheless, despite the high convective velocities, most of the energy in this layer is transported by radiation. The properties of the convective boundary layer obtained using the mixing length theory are shown in Fig. 1.

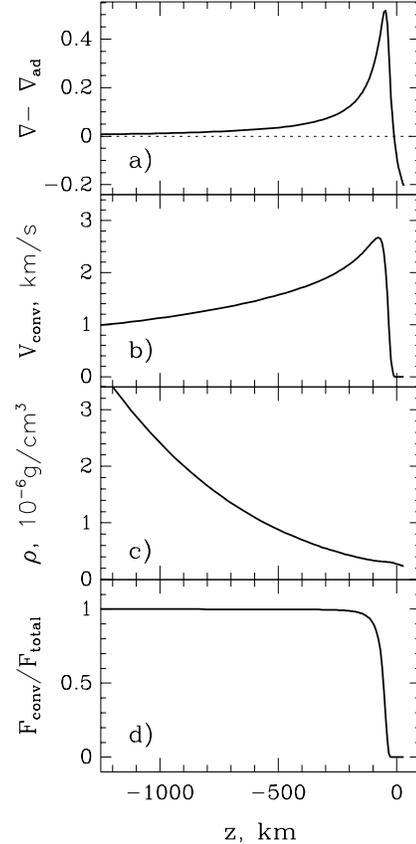


Figure 1: Properties of the upper convective boundary layer computed using mixing length theory: a) the superadiabatic gradient ($\nabla \equiv d \log T / d \log P$); b) convective velocity; c) density; and d) ratio of the energy flux transferred by convection to the total energy flux; z is the height above the photosphere level.

The most characteristic feature of the boundary layer is the sudden change of specific entropy, s , that occurs just beneath the photosphere (Fig.2a). The amplitude of the entropy jump is determined by the overall structure of the Sun and is essentially independent of the details of the boundary transition layer (Gough & Weiss, 1976). However, solar oscillation frequencies are sensitive to the structure of the entropy jump, and therefore provide potential for testing theories of convection.

Despite the significant differences between the mixing-length approach and the picture shown by the numerical simulations, the averaged properties of the layer are qualitatively similar (Spruit, *et al.*, 1990). The mixing-length theory also gives a qualitatively correct estimate of the average vertical velocity. Numerical simulations of the granulation layer have been used to calibrate the mixing length, l , which is usually assumed to be proportional to the pressure scale height, H : $l = \alpha H$, where α is a constant. However, Deupree & Varner (1980), and Atroshchenko & Gadun (1994) found that the mixing-length theory fits the numerical simulations if α varies with depth, being smaller at the very top of the convection zone. They suggested that this decrease of α reflects non-local properties of convection.

One of the important properties for the helioseismic diagnosis of the solar structure is the parameter of convective stability,

$$A^* = \frac{1}{\gamma} \frac{d \log p}{d \log r} - \frac{d \log \rho}{d \log r}, \quad (1)$$

where p is the gas pressure and γ is the adiabatic exponent. The parameter A^* , sometimes called the Schwarzschild discriminant, is positive in convectively stable regions and negative in zones of convection. Its absolute value is very small in most of the solar convection zone, but it experiences a sharp variation with a deep minimum in the upper convective boundary layer (Fig. 2b).

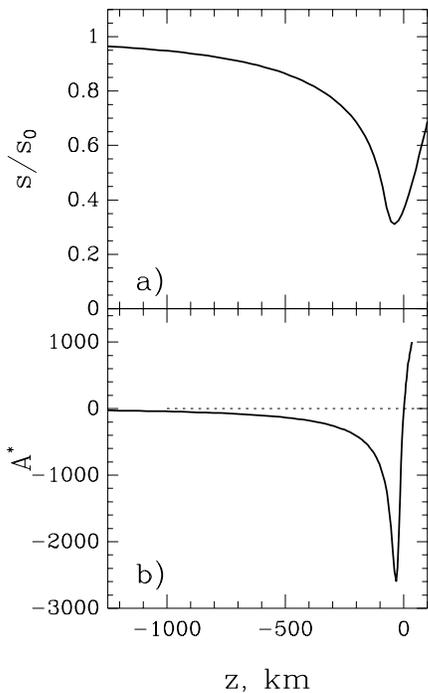


Figure 2: a) The specific entropy, s , and b) the parameter of convective stability, $A^* = ds/d \log r$; s_0 is the specific entropy in the adiabatically stratified part of the convection zone.

The profile of A^* in this layer is very sensitive to details of the convection theory (e.g. Lydon, *et al.*, 1992). Since the definition of A^* includes only the ‘primary’ seismic parameters, p , ρ , and γ , the quantity A^* can be measured seismologically without any other additional assumptions.

2.4. Turbulent pressure

The effects of momentum transfer by convection (turbulent pressure) are often ignored in solar models. However, the turbulent pressure shown in Fig. 3, can be as much as 10% of the total pressure (e.g. Canuto & Mazzitelli, 1991), and therefore, it must be taken into account.

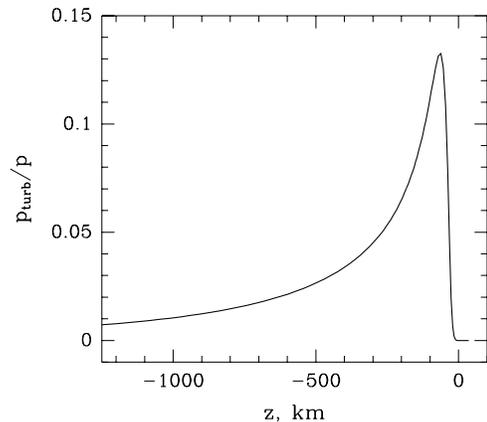


Figure 3: The ratio of the turbulent pressure to the total pressure as a function of height in the upper convective boundary layer

The turbulent pressure affects the structure of the upper convective layer in several ways. It contributes to the total pressure, thus reducing the gas pressure and leading to lower density and sound speed. Turbulent pressure also modifies the superadiabatic gradient, $\nabla - \nabla_{\text{ad}}$ (Heney, *et al.*, 1965), and results in changes in the properties of convection. However, this effect seems to be less significant than the direct impact on the hydrostatic balance (Canuto & Mazzitelli, 1991). Unfortunately, a consistent model with turbulent momentum transfer has not yet been developed, mainly owing to the lack of a turbulence model to provide the turbulence spectrum. However, several different approximations have been used. One of them, suggested by Gough (1977), is based on the Boussinesq approximation and assumes that the turbulence is locally axisymmetric about the radial direction. The mean equation of the hydrostatic balance is then

$$\frac{1}{\rho} \frac{d}{dr} (p + p_{\text{turb}}) + (3 - \Phi) \frac{p_{\text{turb}}}{r\rho} = -\frac{Gm}{r^2}, \quad (2)$$

where p_{turb} is the turbulent pressure given by

$$p_{\text{turb}} = \rho \langle v^2 \rangle, \quad (3)$$

v is the radial component of the convective velocity u_i , and Φ measures the anisotropy of the turbulence:

$$\Phi \equiv \langle u_i u_i \rangle / \langle v^2 \rangle. \quad (4)$$

If the turbulence is isotropic then $\Phi = 3$. The second term in equation (2) represents the net radial force produced by horizontal Reynolds stress for the case of anisotropic turbulence.

The turbulent pressure can be expressed in terms of the mean turbulent speed, V_{conv} :

$$p_{\text{turb}} = \rho \langle v^2 \rangle = \beta \rho V_{\text{conv}}^2, \quad (5)$$

where the constant β depends on the spectrum of the turbulence. Henyey *et al.* (1965) argued that the value of β must be greater than unity because the mean value of the square of the speed is invariably larger than the square of the mean speed. For instance, for a normal distribution of velocity $\beta = \pi/2$. Our preliminary computations of the turbulent pressure effects on p-mode frequencies give a good fit to the observed frequencies if $\beta \approx 3/2$ (see Sec. 3.1).

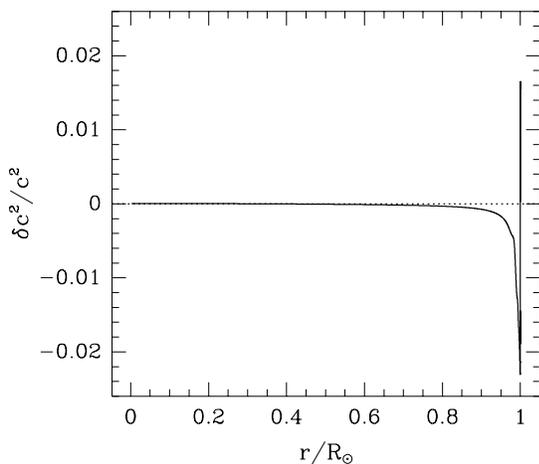


Figure 4: The relative difference of the squared sound speed between the standard solar model with the turbulent pressure and the model without it.

The turbulent pressure decreases the sound speed in the boundary convective layer (Fig. 4) resulting in reduction of frequencies of p modes (Gough, 1984; Balmforth, 1992; Guzik & Cox, 1992). This effect is important for helioseismology and definitely deserves further investigation.

3. EFFECTS OF CONVECTION ON OSCILLATION FREQUENCIES

The upper convective layer affects frequencies of solar oscillations in several ways, which can be separated into two classes: ‘adiabatic’ and ‘non-adiabatic’. The former are due to structure variations in the convective boundary layer; and the latter arise from coupling between oscillations and convection and from absorption and emission of heat as a result of pulsations. Theoretical models of these effects show that the ‘adiabatic’ effects are dominant.

3.1. The effects of structure variation

There are two types of the structure variations in the boundary layer: perturbations of the mean radial stratification, and horizontal inhomogeneity. The spherically symmetrical perturbations shift the mean frequencies of split multiplets, whereas the nonradial inhomogeneities cause variations in frequency splitting within the multiplets (e.g. Lavelly & Ritzwoller, 1992).

The shift of adiabatic eigenfrequencies caused by the radial perturbations can be expressed, using a variational principle, in terms of perturbations of two independent properties of the internal structure, e.g. the parameter of convective stability, A^* , and the adiabatic exponent, γ (see Kosovichev, 1995):

$$I_{nl} \frac{\delta \nu_{nl}}{\nu_{nl}} = \int_0^R K_{A^*}^{(nl)} \delta A^* dr + \int_0^R K_{\gamma}^{(nl)} \delta \gamma dr, \quad (6)$$

where the integral kernels $K_{A^*}^{(nl)}$ and $K_{\gamma}^{(nl)}$ depend on the unperturbed eigenfrequencies, ν_{nl} , and the unperturbed eigenfunctions, $\xi = (\xi_r, \xi_h)$; ξ_r and ξ_h are the radial and horizontal components of the Lagrangian displacement, ξ , of oscillating fluid elements; and

$$I_{nl} = \int_0^R (\xi_r^2 + \xi_h^2) \rho r^2 dr \quad (7)$$

is the moment of inertia of oscillation modes.

Near the surface, the ratio of the horizontal and radial displacements is given by:

$$\frac{\xi_h}{\xi_r} \approx \frac{lg}{\omega_{nl}^2 R} \approx 10^{-3} l \left(\frac{3 \text{ mHz}}{\nu_{nl}} \right)^2, \quad (8)$$

where $\omega_{nl} = 2\pi\nu_{nl}$, and g is the gravitational acceleration. If $l < 10^3$ then the oscillations of the typical frequency 3 mHz are almost radial near the surface. That means that near the surface the kernel functions, $K_{A^*}(r)$ and $K_{\gamma}(r)$, are insensitive to the value of l . Therefore, for a given surface perturbation δA^* and $\delta \gamma$, the right-hand side of Eq. (6) does not depend on l and is approximately a function of frequency ν_{nl} alone. Thus, for a surface perturbation

$$\frac{\delta \nu_{nl}}{\nu_{nl}} \approx \frac{f(\nu_{nl})}{I_{nl}}. \quad (9)$$

In other words, the perturbation of p-mode frequencies due to variations in the structure of the boundary convective layer can be approximately represented by a function frequency alone, scaled with the inverse mode inertia.

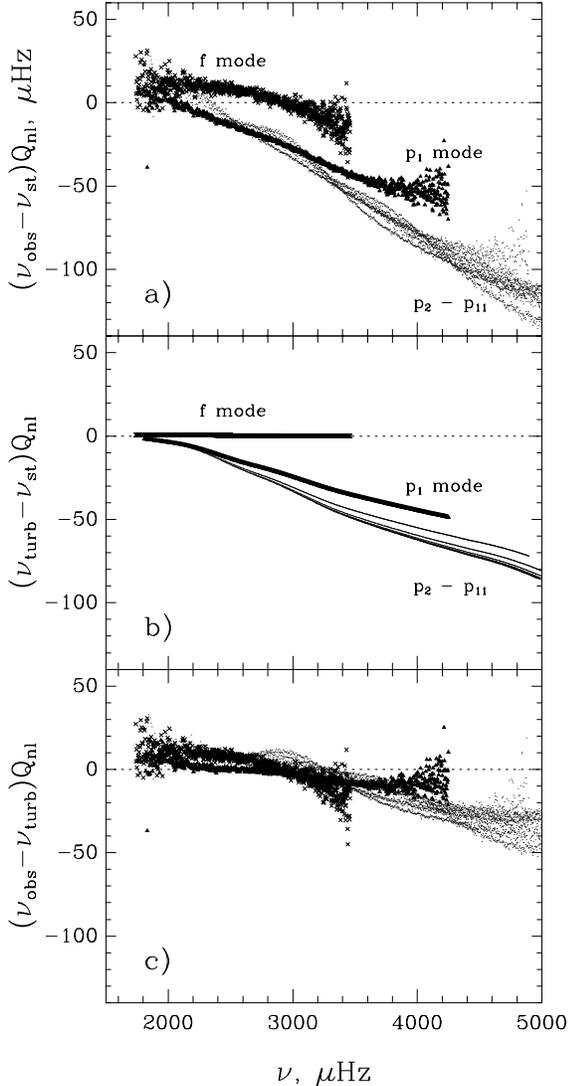


Figure 5: a) The difference between the observed frequencies, ν_{obs} , of high-degree p modes (Bachmann, *et al.*, 1995) and the corresponding frequencies, ν_{st} of a standard solar model computed without effects of turbulent pressure (Christensen-Dalsgaard, *et al.*, 1992). $Q_{nl} = 5I_{nl}/I_{n0}$, where I_{nl} is the moment of mode inertia (Eq. 7). The factor 5 is included to make Q_{nl} of the order of unity.

b) The difference between the frequencies, ν_{turb} , of a solar model in which the effect of turbulent pressure on stratification is taken into account according to Eq. (2) (Kosovichev, in preparation) and the frequencies ν_{st} of the standard model.

c) The frequency difference between the model with the turbulent pressure and the observed frequencies.

This property is commonly used in helioseismic structure inversions to eliminate the uncertainties of the structure of the convective layer from equation (6) (e.g. Basu *et al.*, 1995). The oscillation modes of relatively high angular degree are particularly sensitive to the structure variations near the surface because their mode inertia is relative low. Therefore, they are used to probe the boundary layer.

Fig. 5a shows the scaled difference between the frequencies of the modes of $300 \leq l \leq 1200$ measured by Bachmann *et al.* (1995) and the theoretical frequencies computed for a standard solar model of Christensen-Dalsgaard *et al.* (1992). Fig. 5b shows the difference between the frequencies of a model in which the equilibrium structure was corrected to include the effect of turbulent pressure, and a standard solar model that neglected turbulence effects. The turbulent pressure may account for most of the difference between the observed and model frequencies. It is important to note that the correction of the equilibrium structure of the boundary convective layer does not affect the frequencies of the f mode, which is essentially a surface gravity wave. The mechanisms that can change the frequency of that mode are discussed in Sec. 3.3. The remaining differences between the observed and model frequencies are shown in Fig. 5c. The observed frequencies of p modes are still smaller than their theoretical frequencies; however, the p modes exhibit the same trend as the f mode, suggesting a common mechanism for the frequency variations of both types of oscillations.

The turbulent pressure effect is, perhaps, the main reason why Bachmann *et al.* (1995) found their data in better agreement with the solar model of Guzik & Cox (1992) than with the models provided by Guenther *et al.* (1992) and by Christensen-Dalsgaard *et al.* (1993). It is also likely that for the same reason Paterno *et al.* (1993) found a better agreement with observations when they replaced the standard mixing-length theory without turbulence effects with a new model of convection (Canuto & Mazzitelli, 1991), in which the turbulent pressure is taken into account.

The effects of horizontal inhomogeneities on frequency splitting have not been studied, though a theoretical approach to the effects was suggested by Lively & Ritzwoller (1992). However, no measurements of the frequency splittings of individual multiplets have been made. Such measurements are an important goal for the MDI instrument on board SOHO.

3.2. Turbulent pressure fluctuations, nonadiabatic effects

Variations of frequencies of the oscillation modes may also arise from coupling between convection and oscillations in the convective superadiabatic boundary layer. Such coupling results in fluctuations of the turbulent pres-

sure (Reynolds stresses). It also plays an important role in mode excitation and damping (Sec.4).

Using a time-dependent formulation of mixing-length theory, Gough (1984), Balmforth & Gough (1990) and Balmforth (1992) found that the nonadiabatic effects due to mode interactions with convection and radiation are substantially smaller than the alteration to the equilibrium state caused by turbulent pressure. The effects of nonadiabaticity and turbulent pressure fluctuations become more important at higher oscillation frequencies, because in the convectively active superadiabatic layer and in the radiative atmosphere high- ν modes have relatively greater amplitudes than the modes of low frequency. The result reproduced in Fig. 6 was obtained by Gough & Balmforth (1990) for radial oscillations ($l = 0$). However, the frequency difference is essentially the same for nonradial modes when scaled with the mode inertia I_{nl} .

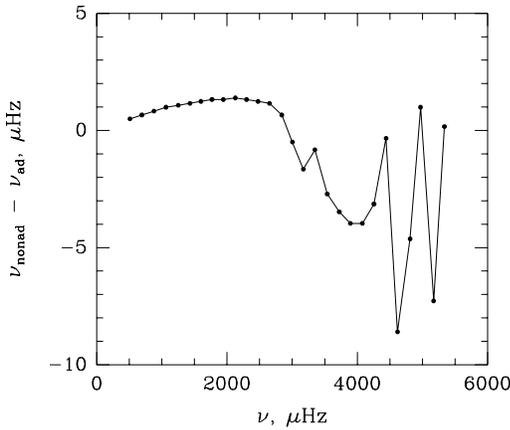


Figure 6: The difference $\nu_{\text{nonad}} - \nu_{\text{ad}}$ (in μHz) between the frequencies ν_{nonad} of nonadiabatic oscillations (in the modified Eddington approximation) including turbulent pressure fluctuations, and the adiabatic eigenfrequencies, ν_{ad} (ignoring turbulent pressure fluctuations) of the same equilibrium model (after Gough & Balmforth, 1990).

The strong fluctuations of the frequency difference at $\nu > 3000 \mu\text{Hz}$ result from turbulent pressure fluctuations. However, Gough & Balmforth (1990) pointed out that since convective conditions in their theory are determined purely locally, the frequency sensitivity of the response of the convection might be artificially enhanced.

Further investigation of these effects with a more reliable description of turbulent convection is necessary, not only because this provides a way to study the physics of the convection, but also because understanding the effects is important for helioseismic inversions. The results by Gough & Balmforth (1990) indicate that it may be incorrect to use the common assumption that the frequency error caused by surface uncertainties can be approximated as a smooth function of frequency (e.g. Basu, *et al.* 1995). For this reason, Däppen *et al.* (1991) and Koso-

vichev *et al.* (1992) recommend using only modes with frequencies below 3 mHz in the helioseismic structure inversions, because these modes do not interact strongly with convection.

3.3. Effects of random velocity fluctuations and the magnetic field

The fluctuating velocity field associated with convection is another factor in the convective boundary layer that decreases the frequency of oscillations. This effect is particularly pronounced for the f mode which is essentially an incompressible surface wave, insensitive to both structure variations (see Fig. 5b) and nonadiabatic effects (e.g. Zhugzhda, 1982). However, there is systematic difference between the observed and theoretical frequencies of the f mode (Fig. 5a). The observed frequencies are somewhat higher than theory predicts at low ν , but become lower as the frequency (or the degree) increases. Such behaviour of the f-mode frequencies can be understood in terms of the combined action of a magnetic field and random convective flows (Murawski & Roberts, 1993a,b; Murawski & Goossens, 1993).

A magnetic field exerts an additional tension force on this surface mode. As a consequence the phase speed of the wave is larger in the presence of the magnetic field, and this leads to an increase of the oscillation frequency for a given wave number. On the other hand, random convective flows can decrease the frequency, because a wave takes longer to travel a given distance due to the scattering from random inhomogeneities (Howe, 1971).

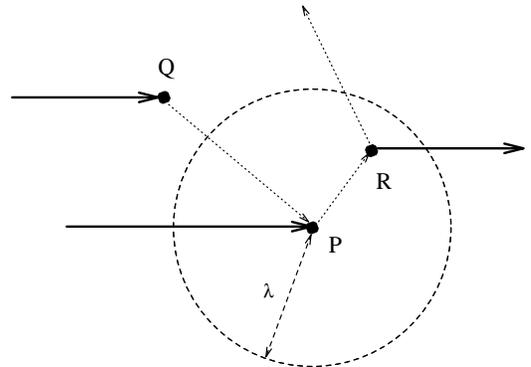


Figure 7: Wave scattering from random inhomogeneities. Dotted lines represent scattered waves; full lines represent components of the coherent field; λ is the correlation scale of the fluctuations (after Howe, 1971).

The wave scattered from a point, P, will be scattered again at some other point, R (Fig. 7). Such a collision will produce a scattered random field together with a mean field component. The latter represents the energy scattered back into the coherent wave field, and will be significant

if the distance between P and R is less than the correlation length, λ , of the random fluctuations. As a result of the scattering, the frequency of a wave is reduced by the order of the mean squared amplitude of the fluctuations, and the energy of the coherent wave is gradually absorbed.

Murawski & Goossens (1993) demonstrated that a model with random convective flows and a chromospheric magnetic field can fit the observed f-mode frequencies. It is likely that frequencies of p modes are also reduced by the same mechanism, explaining the remaining difference between theory and observations (Fig. 5c). However, the computations for p modes have not been carried out.

Finally, we note that the fine difference between p-mode ridges in Figs 5a and 5c is likely to be due to small variations of the mean stratification of the convective boundary layer. Such structure variations, which can be, in principle, inferred by frequency inversion, are of interest for testing theories of solar convection in greater detail.

4. MODE EXCITATION AND DAMPING

It is generally believed that f and p modes of solar oscillation are excited randomly by the turbulence in the upper convective boundary layer. This belief is based mainly on theoretical models showing that the modes are most likely to be intrinsically stable (e.g. Balmforth, 1992). However, the physics of interaction between oscillations and convection, the key element of the models, is poorly understood, and the theory of mode excitation and damping is far from being in quantitative agreement with observations.

4.1. Intrinsic stability

The vibrational stability of the Sun has a long history of investigation (see Balmforth, 1992, and references therein). The main non-adiabatic process that contributes to destabilization of the p modes is related to variations of opacity in the hydrogen ionization zone and represents a generalized form of the conventional κ -mechanism. The κ -mechanism which operates in classical pulsating stars works, in principle, on modulation of the radiative energy flux. When, due to oscillation, the outer layer of a star is compressed, the degree of ionization increases. This leads to an increase of opacity and to absorption of a greater amount of heat than would have occurred without oscillations. In the subsequent expansion, the extra heat stored in the ionization zone is converted to mechanical work, thus amplifying the oscillation. However, in the Sun, the zones of ionization lie in the convection zone where the amount of heat transported by radiation is relatively small. Therefore, this mechanism can provide rather weak driving.

The detailed driving or damping of the solar acoustic modes largely depends on their interaction with convection. Oscillations perturb the turbulent fluxes of heat and momentum. How these perturbations in turn influence the oscillations crucially determines their intrinsic stability. A mathematical description of these processes must involve a theory of turbulence in the upper convective layer. So far, a detailed investigation was carried out only for a time-dependent, non-local mixing-length theory (Gough & Balmforth, 1990; Balmforth, 1992).

The time-dependent theory of convection determines how oscillations perturb the turbulent flux and momentum. The perturbations then react back on the processes of mode driving and damping. The coupling between the oscillations and convection takes two principal forms: thermal effects due to modulation of the convective heat flux and dynamical effects produced by perturbations of the turbulent pressure (Reynolds stresses). The computations by Gough and Balmforth showed that the perturbations of the turbulent pressure are principally responsible for stabilizing the oscillations. The non-adiabatic processes related to opacity changes due to oscillations (the " κ -mechanism") compete with the damping but are rather weaker. The contributions of these processes to the damping rate, η , are shown in Fig. 8.

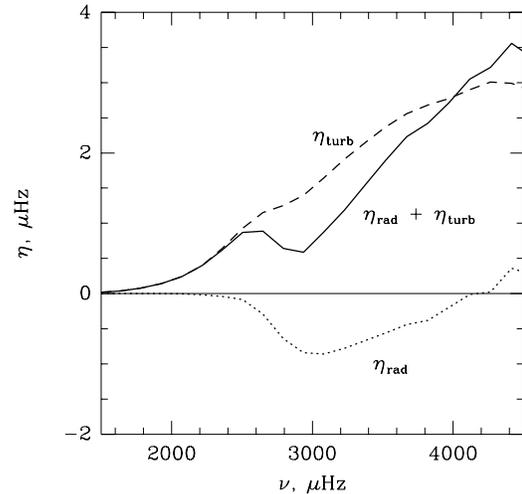


Figure 8: The damping rate η (solid curve) of radial acoustic modes computed by Balmforth (1990) for a time-dependent mixing-length theory of convection. The dotted curve shows the contribution to η due to interaction between oscillations and radiation; the dashed line shows the contribution of turbulent pressure fluctuations.

Therefore, the theory predicts that solar p modes are intrinsically stable. The theoretical estimate of the damping rate is in qualitative agreement with the measurements of the oscillation line width (Fig. 9).

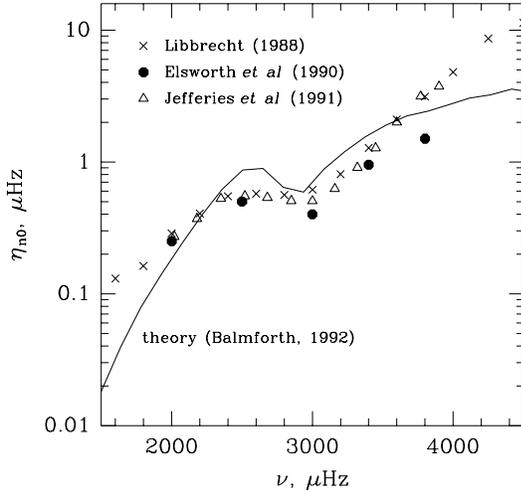


Figure 9: *The observed and theoretical damping rates η of the radial oscillations ($l = 0$) as a function of frequency. The measurements of linewidth ($\Gamma \equiv 2\eta$) by Libbrecht (1988) (crosses) of $l = 20$ modes and by Jefferies et al. (1991) (triangles) of $l = 100$ modes are scaled with the factor $I_0(\omega)/I_l(\omega)$ (inverse relative mode inertia) to account for the l -dependence of the line widths. The theoretical values (solid curve) were obtained by Balmforth (1992) using a time-dependent mixing-length theory*

The theory explains the characteristic depression of η ('plateau') observed at 2800 μHz by the contribution of the " κ -mechanism" (see Fig. 8). However, it disagrees with the observations at low and high frequencies. We should note that there is also significant uncertainty in current linewidth measurements which may be caused by gaps in ground-based observations and by other factors (see Harvey, these proceedings). Therefore, reliable measurement of the line widths of oscillations is an important goal of the GONG and SOI projects.

4.2. Stochastic excitation

It is likely that acoustic radiation emitted by turbulent flows in the upper convective boundary layer is the primary source of solar five-minute oscillations. Generation of sound by turbulence is normally described by an inhomogeneous wave equation, in which the inhomogeneous term represents the random body force arising from convective entropy fluctuations and turbulent stresses (Lighthill, 1952; Stein, 1967; Goldreich & Keely, 1977).

In such a model, mode amplitudes are described by the equation of a damped harmonic oscillator:

$$I_{nl} \left(\frac{d^2 a_{nl}}{dt^2} + 2\eta_{nl} \frac{da_{nl}}{dt} + \omega_{nl}^2 a_{nl} \right) = F(t), \quad (10)$$

where a_{nl} is a measure of the surface displacement of a mode of angular degree l and radial order n ; ω_{nl} , I_{nl} and η_{nl} are the frequency, inertia, and damping rate; and $F(t)$ is a random forcing function. If $\eta_{nl} \ll \omega_{nl}$ then the power spectrum, P_a , of a mode is given by:

$$P_a(\omega) \propto P_L(\omega)P_F(\omega) \equiv \frac{1}{(\omega^2 - \omega_{nl})^2 + \eta_{nl}^2} P_F(\omega), \quad (11)$$

where $P_L = 1/[(\omega^2 - \omega_{nl})^2 + \eta_{nl}^2]$ is a Lorentzian profile with the half-width $\Gamma_{nl} = 2\eta_{nl}$ and $P_F(\omega)$ is the power spectrum of F . Fig. 10 shows an example of the power spectrum of an oscillation mode.

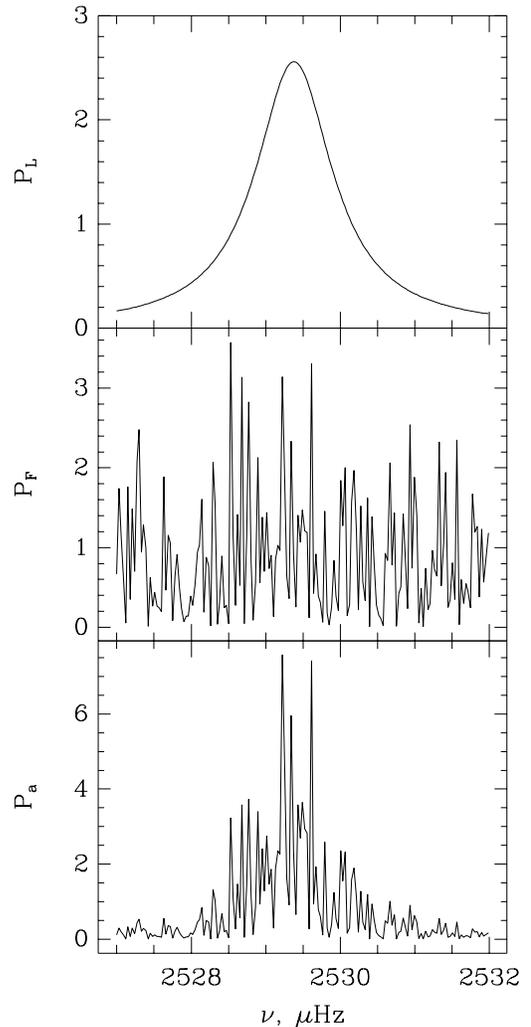


Figure 10: *Illustration of the effect of random excitation on a spectral line in the oscillation power spectra. P_L is a pure Lorentzian representing the p mode with $l = 20, n = 11$ ($\nu = 2529.378 \mu\text{Hz}$, $\Gamma = 1.25 \mu\text{Hz}$); P_F is a stochastic source spectrum; and P_a is the resultant spectrum of the mode. P_a is the product of the Lorentzian and the source spectrum. The sampling rate is 0.03 μHz*

The spectrum of the stochastic force appears in the oscillation power spectrum as multiplicative noise and significantly complicates the solution of the basic helioseismic problem – estimation of the spectral line parameters, such as line widths, central frequencies and amplitudes. Keely and Ritzwoller (1993) and Gough *et al.* (1995) suggested estimating smooth trends of the parameters over a large number of spectral lines in order to reduce the uncertainty due to the multiplicative noise. Another approach, which is being developed by Chang and Gough (1995), is to include properties of stochastic excitation in the estimation of spectral parameters.

The total energy $E_{nl} = \frac{1}{2}I_{nl}(\dot{a}_{nl}^2 + \omega_{nl}^2 a_{nl}^2)$ associated with an oscillation mode satisfies

$$\frac{dE_{nl}}{dt} + \Gamma_{nl}E_{nl} = G_{nl}, \quad (12)$$

where $G_{nl} = \langle \dot{a}_{nl}F \rangle$ is the power of the driving force, averaged over a period of oscillation. If a mode is excited stochastically then its energy fluctuates on the damping time scale and has the mean value

$$\overline{E}_{nl} = \frac{G_{nl}}{\Gamma_{nl}}. \quad (13)$$

Assuming that $F(t)$ is a Gaussian random process, Kumar *et al.* (1988) calculated the probability distribution functions for the energy, E_{nl}^T , averaged over a time interval, $-T \leq t \leq T$. If $T\Gamma_{nl} \ll 1$, then the distribution is obviously a decaying exponential:

$$p(E_{nl}^T) = \exp\left(-\frac{E_{nl}^T}{\overline{E}_{nl}}\right). \quad (14)$$

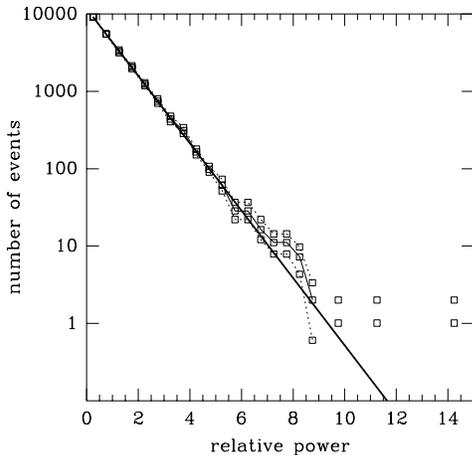


Figure 11: Distribution of mode power compared with prediction. The points represent the distribution function of half-day estimates of mode power relative to mean mode power obtained from the BISON data in 1987-1994 by Elsworth *et al.* The thick solid line is the theoretical prediction by Kumar, *et al.* (1988) based on a stochastic mechanism of mode excitation (after Elsworth *et al.*, these proceedings).

Fig. 11 shows the distribution of the mode energy averaged over a half day ($T\Gamma \simeq 10^{-2}$) determined from observations by Elsworth *et al.* (these proceedings) as compared with the prediction (Eq. 14).

It is clear that the observed and theoretical distributions are in good agreement (except, perhaps, at the high-energy end), giving a strong argument that the mechanism of mode excitation is indeed stochastic.

The theories of stochastic excitation (Stein, 1967; Goldreich & Keeley, 1977; Osaki, 1990; Balmforth, 1992; Goldreich *et al.*, 1994; Musielak *et al.*, 1994) evaluate the turbulent power input, G_{nl} . The estimates of the input are uncertain because they depend upon details of the model of turbulence used in the theories. Fig. 12 shows the mean mode amplitude computed by Balmforth (1992) using time-dependent non-local mixing-length theory and assuming that the turbulence in the convection zone is homogeneous with a Kolmogorov spectrum.

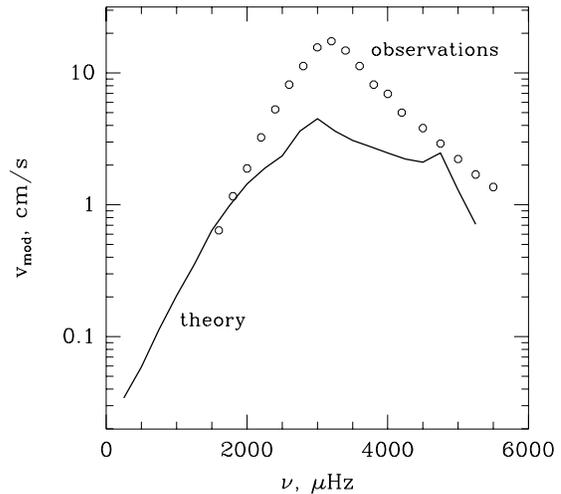


Figure 12: Mean mode amplitudes measured by Libbrecht (1988) (circles) and predicted by Balmforth (1992) using a Kolmogorov spectrum of convective turbulence (solid curve) (after Balmforth, 1992).

The theoretical estimates are substantially lower than the observed mode amplitudes shown in the same figure. Musielak *et al.* (1994) found that the theoretical results do not change significantly for other plausible spectra of the turbulence in the convection zone. There were also several attempts to bring the theory into agreement with observations by scaling the linewidth Γ_{nl} in Eq.(13) (see Gough & Toomre, 1991, for a discussion).

However, there is evidence that substantial acoustic power is generated in isolated acoustic events, probably associated with exploding granules and mesogranulation, which are not described by the standard theories of stochastic excitation (Brown, 1991; Goode, *et al.*, 1992; Restanio, *et al.*, 1993). Simultaneous high-resolution observations of granulation and acoustic events by Rimmele *et al.*

(1995) found that the acoustic flux occurs preferentially in dark lanes of granulation. The lanes abruptly darken immediately before the acoustic power of the events peaks (Fig. 13).

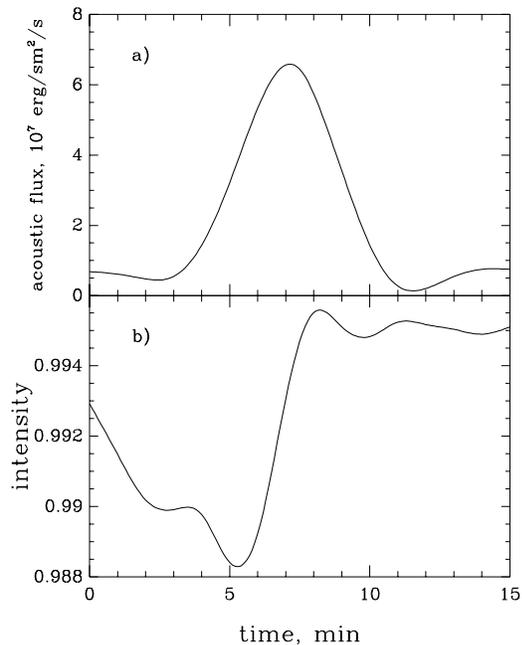


Figure 13: Time evolution of (a) acoustic flux and (b) normalized granular intensity averaged over several hundred acoustic events (Rimmele, et al., 1995)

The dark lanes observed in granulation are probably associated with rapid cooling and strong downflows that develop in the centres of exploding granules. Numerical simulation by Stein & Nordlund (1990) and by Rust & Toomre (1993) have demonstrated the fast downward-plunging cool plumes are likely sites of significant acoustic excitation.

5. CONCLUSIONS

The upper convective boundary layer, which is only a few hundred km thick, is a subphotospheric region of superadiabatic temperature gradient, where convection takes the form of extremely unstable turbulent eddies (granules) and is characterized by large-amplitude fluctuations of the thermodynamic state and nearly sonic velocities. It is the place in the Sun where Reynolds stresses and nonadiabatic effects have their greatest influence on the frequencies and stability of five-minute oscillations. A reliable theory of the interaction of the oscillations with the turbulent layer has yet to be developed. Helioseismology provides a valuable tool for studying the interaction and also for uncovering errors made in the standard solar model due to poor understanding of the structure and dynamics of the

layer. Such studies are of great importance for improving the accuracy of diagnostics of the Sun's deep interior.

Accurate measurements of high-degree f- and p-mode frequencies, such as those expected from the MDI instrument on board SOHO, can be used to determine the average properties of the superadiabatic layer: the radial gradient of the specific entropy and the turbulent pressure. These properties, inferred directly from oscillation frequencies using an inversion procedure, can be used for testing and calibrating convection theories and 3D numerical simulations.

The turbulent pressure (Reynolds stresses), often neglected in the standard solar model, significantly affects the equilibrium stratification of the convective boundary layer and is responsible for most of the difference between the observed and theoretical frequencies of high-degree p modes. Therefore, the turbulent pressure effects must be taken into account in future realizations of the standard solar model, though a consistent approach to these effects has yet to be developed.

The remaining frequency residuals are probably due to the interaction between the oscillations and random fluctuations of velocity, density and magnetic field in the granulation layer. These effects are most apparent in the residuals of frequencies of the f mode, which is not sensitive to variations in the mean hydrostatic stratification. Therefore, accurate measurements of f-mode frequencies (and their temporal variation) over a broad range of horizontal wavenumbers is important for studying effects of the convective inhomogeneities and the magnetic field.

The upper convective boundary layer is likely to be the place where five-minute oscillations are excited. There is observational and theoretical evidence that the oscillation modes are intrinsically stable and that they are excited stochastically due to turbulent fluctuations of entropy and Reynolds stresses. The most advanced theory of mode excitation and damping, based on a time-dependent non-local mixing-length approach, predicts mode linewidths and amplitudes that are in a qualitative, but not quantitative, agreement with observations. It is an important goal for both the GONG and SOI-MDI projects to measure the linewidths of oscillation modes reliably.

The distribution of mode energy fits well with theoretical prediction based on the idea of stochastic excitation. However, the theoretical estimates of mode amplitudes are substantially lower than those measured. There is evidence that substantial oscillation power is generated in isolated acoustic events associated with dark lanes in the granulation pattern, which are probably associated with exploding granules. Such acoustic events are not described by the theories of stochastic excitation, and need to be studied both observationally and theoretically.

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REFERENCES

1. Atroshchenko, I.N., and Gadun, A.S., 1994, *Astron. Astrophys.*, **291**, 635
2. Bachmann, K.T., Duvall, T.L., Jr., Harvey, J.W., and Hill, F., 1995, *Astrophys. J.*, **443**, 837
3. Balmforth, N.J. 1992, *Mon. Not. R. astr. Soc.*, **255**, 603
4. Balmforth, N.J., and Gough, D.O. 1990, *Solar Phys.*, **128**, 161
5. Basu, S., Thompson, M.J., Christensen-Dalsgaard, J., and Hernández, F.P., 1995, in Proc. GONG'94 Conf. *Helio- and Asteroseismology from the Earth and Space*, eds R. Ulrich, E. Rhodes and W. Däppen, Astron. Soc. Pac., San Francisco, p.152
6. Böhm-Vitense, E. 1958, *Z. Astrophys.*, **46**, 108
7. Brown, T.M., 1991, *Astrophys. J.*, **371**, 396
8. Canuto, V.M., and Mazzitelli, I., 1991, *Astrophys. J.*, **370**, 295
9. Cattaneo, F., Brummell, N.H., Toomre, J., Malagoli, A., and Hurlburt, N.E. 1991, *Astrophys. J.*, **370**, 282
10. Chang, H.-Y., and Gough, D.O., 1993, in: *GONG 1992: Seismic Investigation of the Sun and Stars*, ed. T.M. Brown, Astron. Soc. Pac., San Francisco, p. 457
11. Christensen-Dalsgaard, J., Proffitt, C.R., and Thompson, M.J. 1993, *Astrophys. J.*, **403**, L75
12. Cox, J.P., and Giuli, R.T. 1968, *Principles of Stellar Structure*, Gordon and Breach, New York
13. Däppen, W., Gough, D.O., Kosovichev, A.G., and Thompson, M.J. 1991, in *Challenges to Theories of the Structure of Moderate-Mass Stars, Lecture Notes in Physics*, **388**, p. 111
14. Deupree, R.G., and Varner, T.M., 1980, *Astrophys. J.*, **237**, 558
15. Elsworth, Y., Isaak, G.R., Jefferies, S.M., McLeod, C.P., New, R., Pallé, P.L., Régulo, C., and Roca Cortés, T. 1990, *Mon. Not. Roy. Astron. Soc.*, **242**, 135
16. Gilbert, J.F. 1971, *Geophys. J. R. astron. Soc.*, **23**, 119
17. Goldreich, P., and Keeley, D.A. 1977, *Astrophys. J.*, **212**, 243
18. Goldreich, P., Murray, N., and Kumar, P. 1994, *Astrophys. J.*, **424**, 466
19. Goode, P.R., Gough, D.O., and Kosovichev, A.G. 1992, *Astrophys. J.*, **387**, 707
20. Gough, D.O., 1977, *Astrophys. J.*, **214**, 196
21. Gough, D.O. 1984, *Adv. Space Res.*, **4(8)**, 85
22. Gough, D.O., Kosovichev, A.G., and Toutain, T., 1995, *Solar Phys.*, **157**, 1
23. Gough, D.O., and Toomre, J. 1991, *Ann. Rev. Astron. Astrophys.*, **29**, 627
24. Gough, D.O., and Weiss, N.O., 1976, *Mon. Not. Roy. astron. Soc.*, **176**, 589
25. Guenther, D.B., Demarque, P., Kim, Y.C., and Pinsonneault, M.H. 1992, *Astrophys. J.*, **387**, 372
26. Guzik, J.A., and Cox, A.N. 1992, *Astrophys. J.*, **386**, 729
27. Henyey, L., Vardya, M.S., and Bodenheimer, P., 1965, *Astrophys. J.*, **142**, 841
28. Howe, M.S., 1971, *J. Fluid Mech.*, **45**, 769
29. Jefferies, S.M., Duvall, T.L., Harvey, J.W., Osaki, Y., and Pomerantz, M.A. 1991, *Astrophys. J.*, **377**, 330
30. Kelly, J.F., and Ritzwoller, M.H. 1993, *Astrophys. J.*, **418**, 476
31. Kosovichev, A.G., 1995, in Proc. GONG'94 Conf. *Helio- and Asteroseismology from the Earth and Space*, eds R. Ulrich, E. Rhodes and W. Däppen, Astron. Soc. Pac., San Francisco, p.89
32. Kosovichev, A.G., and Perdang, J. 1988, in *Seismology of the Sun and Sun-Like Stars*, ed. E.J. Rolfe, Paris: ESA SP-286, 539
33. Kosovichev, A.G., Christensen-Dalsgaard, J., Däppen, W., Dziembowski, W.A., Gough, D.O., and Thompson, M.J. 1992 *Mon. Not. Roy. astron. Soc.*, **259**, 536
34. Kumar, P., Franklin, J., and Goldreich, P. 1988, *Astrophys. J.*, **328**, 879

35. Lively, E.M., and Ritzwoller, M.H., 1992, *Phil. Trans. R. Soc. Lond. A*, **339**, 413
36. Libbrecht, K.G. 1988, *Astrophys. J.*, **334**, 510
37. Lighthill, M.J. 1952, *Proc. Roy. Soc. London*, **A211**, 564
38. Lydon, T.J., Fox, P.A., and Sofia, S., 1992, *Astrophys. J.*, **397**, 702
39. Murawski, K., and Roberts, B. 1993a, *Astron. Astrophys.*, **272**, 595
40. Murawski, K., and Roberts, B. 1993b, *Astron. Astrophys.*, **272**, 601
41. Murawski, K., and Goossens, M. 1993, *Astron. Astrophys.*, **279**, 225
42. Musielak, Z.E., Rosner, R., Stein, R.F., and Ulmschneider, P., 1994, *Astrophys. J.*, **423**, 474
43. Nordlund, A. and Dravins, D. 1990, *Astron. Astrophys.*, **228**, 155
44. Osaki, Y., 1990, in *Progress of Seismology of the Sun and Stars*, eds Y.Osaki and H.Shibahashi, Berlin, Springer-Verlag, p.75
45. Paterno, L., Ventura, R., Canuto, V.M., and Mazzitelli, I. 1993 *Astrophys. J.*, **402**, 733
46. Rast, M.P., Nordlund, A., Stein, R.F., and Toomre, J., 1993, *Astrophys. J. Lett.*, **408**, L53
47. Rast, M.P., and Toomre, J., 1993, *Astrophys. J.*, **419**, 240
48. Rast, M.P., 1995, *Astrophys. J.*, **443**, 863
49. Rimmele, T.R., Goode, P.R., Harold, E., and Stebbins, R.T., 1995, *Astrophys. J. Lett.*, **444**, L119
50. Restanio, S.R., Stebbins, R.T., and Goode, P.R. 1993, *Astrophys. J. Lett.*, **408**, L57
51. Spruit, H.C., Nordlund, A., and Title, A.M. 1990, *Ann. Rev. Astron. Astrophys.*, **28**, 263
52. Stein, R.F., 1967, *Solar Phys.*, **2**, 385
53. Stein, R.F., and Nordlund, A., 1990, in *Progress of Seismology of the Sun and Stars*, eds Y.Osaki and H.Shibahashi, Berlin, Springer-Verlag, p.93
54. Title, A.M., Tarbell, T.D., Topka, K.P., Ferguson, S.H., Shine, R.A., and the SOUP Team, 1989, *Astrophys. J.*, **336**, 475
55. Zhugzhda, Y.D. 1982, *Sov. Astron. Lett.*, **8**, 302