INTERIOR STRUCTURE & INVERSIONS
(Working Group Report)

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ABSTRACT

The paper contains the contributions reported at the Working Group Session 'Interior Structure and Inversions'. Discussion at this session has been focused on problems of inversion analyses of current solar oscillation data and future data from the SOHO and GONG projects. New developments in helioseismological inversion techniques and in modeling physical processes in the Sun's interior have been presented.

Keywords: *Sun's interior, oscillations, data inversion, equation of state, opacity, solar models, SOHO*

INTRODUCTION

Accurate determination of the properties of the Sun's interior and diagnosis of the physical processes inside the Sun are of fundamental importance for many branches of astrophysics and physics. These studies provide an important test for the theory of stellar structure and evolution, shed light on formation of the solar system and its chemical evolution, and help to understand the mechanism of solar and stellar activity. They also allow study of physical processes in a dense plasmas under conditions impossible to achieve in laboratories.

These problems will be studied by inverting frequencies of normal modes of solar oscillation, accurate measurements of which will be provided by both the Global Oscillation Network Group (GONG) (Leibacher, these Proceedings) and the Solar Oscillation Investigation (SOI) (Scherrer, these Proceedings). The latter is the largest helioseismology project on board SOHO. It uses the Michelson Doppler Imager (MDI) instrument to measure intermediate- and high-degree modes of solar oscillations. The two other helioseismology projects on SOHO, GOLF (Gabriel, these Proceedings) and VIRGO (Fröhlich & Andersen, these Proceedings), are devoted to observations of modes of low angular degree.

The purpose of the Working Group was to discuss recent developments in helioseismic inversions and in modeling the solar structure in preparation for analysis of data from the SOHO instruments. The contribution by Kosovichev (Sec. 1) outlines the basic objectives and the inversion procedure of the SOI Structure Inversion Program. The procedure is based on two levels of analysis: 'primary' inversions for hydrostatic properties of the Sun (pressure, density and the adiabatic exponent), and 'secondary' inversions for chemical abundances, temperature and other properties, determination of which involves additional physical constraints such as the equation of state (EOS) and the radiative opacity. The progress in modeling the equation of state and the opacity has been reported by Rogers and Iglesias in Sec. 2. They have calculated new opacity and EOS tables including effects of seven additional heavy elements. These accurate tables are important for the secondary inversions and for modeling the solar structure and evolution. Christensen-Dalsgaard has presented a new evolutionary solar model computed using the most recent physical parameters (Sec. 3). He has pointed out that the improved physics bring the model into better agreement with the inversions, although there remain significant differences, particularly in the solar core. The contribution by œ (Sec. 4) deals with determination of the hydrogen abundance and temperature in the radiative interior using asymptotic sound-speed inversion. This technique is complementary to the approach presented by

Kosovichev, and, therefore, it is important for independent estimates of the 'secondary' solar parameters. The last three contributions concern with inversion techniques. Basu (Sec. 5) has argued that inverting combinations of oscillation frequencies taken from different data sets may result in systematic errors in the inversion results, particularly, if the data were obtained at different phases of solar activity. She has also emphasized importance of accurate measurements of high-degree modes along with the ones of low degree in order to make robust estimates of properties of the solar core. Gough and Sekii (Sec. 6) has pointed out that reliable determination of the covariance matrix of errors in the frequency measurements is crucial for accuracy of the linear inversions. Eff-Darwich and Pérez-Hernández (Sec. 7) has studied how measurements of low-frequency p and g modes, expected from GOLF and VIRGO data, can affect determination of the radial hydrostatic structure of the Sun. They have demonstrated that accuracy of the inversions is significantly improved even if only few low-frequency modes are added to a data set.

1. SOI STRUCTURE INVERSION PROGRAM

by A.G. Kosovichev

The SOHO and GONG data will be unique, providing continuous observations of solar oscillations for several years. This will allow study of temporal variations of the Sun's properties and the relations of these variations to the solar activity cycle. It is also important for the inversion that the projects will provide simultaneous data for modes of both low and intermediate angular degree, l, in the range from 0 to approximately 200. This is essential for probing the deep interior of the Sun, particularly for obtaining helioseismic constraints on the properties of the energy-generating core. New helioseismic inversion techniques will be developed in order to fully utilize the potential of the new data.

Current knowledge of the internal structure of the Sun was obtained using relatively short (a few months) interrupted series of observations, mainly, at the Big Bear Solar Observatory and at the South Pole. These results have provided important constraints on the internal properties and on the physical processes in the Sun. However, many fundamental questions are still unanswered, e.g.

- What are the physical conditions (temperature, density, chemical composition) in the energy-generating core?
- Is there any deviation from the conditions of thermal equilibrium in the solar core?
- Is there evidence of localized material mixing in the core?
- What is the role of gravitational settling and turbulent diffusion of helium and heavy elements in the radiative zone?
- How deep is the zone of convective overshoot at the lower boundary of the convection zone?
- Is there latitudinal variation of the depth of the convection zone?
- What is the helium abundance in the convection zone?
- How accurate are current models of microscopic physics: the equation of state, opacities and nuclear reaction rates, in the Sun?
- What is the structure of the surface boundary layer and how does it interact with the oscillations?

These questions form the core of the program. To fully exploit the large amount of high-precision SOHO and GONG data it is necessary to perform special studies of inversion methods including modifications of existing techniques in order to achieve greater spatial resolution and accuracy of inverted parameters.

The results of the program will be development of the helioseismic inversion techniques; measurements of the properties of the solar core (density, sound speed, temperature, chemical composition); determination of the distribution of helium and heavy elements in the radiative zone and the role of gravitational settling and diffusion in the distribution; estimation of the parameters of the convective overshoot zone and its role in dynamic processes in the Sun; determination of the helium abundance at the solar surface, that is not measurable by spectroscopic techniques; assessment of accuracy of stellar evolution theory.

The principal inversion procedure is based on a perturbation theory derived from a variational principal for adiabatic stellar pulsations and is set up to measure deviations of seismic parameters from a spherically-symmetrical reference solar model. Our initial analysis of the data will restrict these studies to axisymmetric deviations from the model, the axis of symmetry coinciding with the axis of solar rotation. The axisymmetric perturbations can be obtained by inverting averages over groups of modes; this significantly reduces both the computational time and the formal errors of the inversion results. These inversions will be done on a regular basis (say, monthly), in the data analysis 'pipeline'. Important experience has been gained from determination of the axisymmetric structure by the inversions of the BBSO and other ground-based data (Antia & Basu, 1994; Basu, *et al.*, 1995; Däppen, *et al.*, 1990; Dziembowski, *et al.*, 1990; Gough & Kosovichev, 1988, 1990; Gough, Kosovichev & Toutain, 1995; Kosovichev & Fedorova, 1991; Kosovichev, *et al.*, 1992; Kosovichev, 1993, 1995a,b)

Capability for studying nonaxisymmetric perturbations seismologically will be sensitive to the quality of the GONG and SOI data. However, it is planned to study approaches to non-axisymmetric inversions, e.g. by fitting simple models to the data (cf. Kosovichev & Perdang, 1988). Studies of the non-axisymmetric perturbations inside the Sun are of particular interest for understanding mechanisms for the solar cycle and its relation to the structure of large-scale convection. However, most questions addressed in this program deal with the spherically-symmetric component of the deviation from the standard solar model. These deviations are determined from the shifts of frequencies of the axisymmetric modes with respect to model frequencies, or from the shifts of the mean frequencies of mode multiplets.

For inversions of this type the linearized integral equations relating frequency perturbations $\delta\nu_i$, to variations of the solar structure are derived from a variational principle. The equations are transformed to depend on a chosen pair of deviation variables (δf and δg) that are assumed to be functions of radius r alone (e.g. Kosovichev, 1992), yielding

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{\mathrm{R}_\odot} \left[K_{f,g}\frac{\delta f}{f} + K_{g,f}\frac{\delta g}{g}\right] \mathrm{d}r + \frac{F(\nu_i)}{E_l(\nu_i)}, \quad i = 1, ..., N, \qquad (1)$$

where $\delta\nu_i$ is the frequency difference between the eigenfrequency, ν_i, of a solar model and the corresponding frequency of the Sun, $f(r)$ and $g(r)$ are structure parameters, $\mathrm{R}_\odot$ is the radius of the Sun, and $E_l(\nu_i)$ is the mode inertia. The arbitrary function $F(\nu_i)$ is added to take into account the surface effects. The suffix i labels the modes; N is the total number of modes in a data set.

The structure parameters, f and g, can be of two types: 'primary', e.g. the density, ρ, and the adiabatic exponent, γ, or 'secondary', e.g. the hydrogen abundance, X, and the heavy element abundance, Z. For the 'primary' parameters Eq. (1) is derived using only basic assumptions about solar structure: spherical symmetry and hydrostatic equilibrium; whereas additional structure equations have to be considered for the 'secondary' parameters. Two main options for the 'secondary' parameters have been studied. The first uses the equation of state in the form $\gamma = \gamma(p, \rho, X_j)$, where X_j are element abundances. Since variations of the adiabatic exponent, γ, occur mainly in the ionization zones of helium and hydrogen at the top of the convection zone, the frequency variations can be expressed in terms of the uniform helium abundance in the convection zone (assuming that the abundances of the heavier elements are known) and one of the hydrostatic variables, e.g. density or the ratio $u = p/\rho$ (cf Dziembowski, *et al.*, 1990).

The second option considers a full set of structure equations including thermal balance and energy transport. This introduces 'non-seismic' variables, such as temperature, element abundances, and radiative zone opacities into the seismic equations (Gough & Kosovichev, 1988; 1990).

Two basic inversion procedures, Optimally Localized Averaging (OLA) and Regularized Least Squares (RLS) data fitting, have been developed for the structure inversion pipeline. Both methods essentially subtract from the $\delta\nu/\nu$ data a quantity $F(\nu)/E_l$ describing unknown surface effects. The OLA technique will be most commonly used in the project. It consists of constructing linear combinations of Eq. (1) for a set of observed modes that provide localized averages of the structure parameters f and g (e.g. Backus & Gilbert, 1967):

$$\overline{\frac{\delta f}{f}}(r_0) = \int_0^{\mathrm{R}_\odot} A_{f,g}(r_0, r)\frac{\delta f}{f}\mathrm{d}r \qquad (2)$$

in the vicinity of $r = r_0$. The spatial resolution of the averages can be characterized by the central coordinate and the resolution scale of the averaging kernels. These quantities, 'centre' and 'spread', usually represent some integral measure of the kernels in order to account for effects asymmetry and sidelobes.

Selections of the localized averaging kernels for u, obtained for a subset of solar data, are shown in Fig. 1.

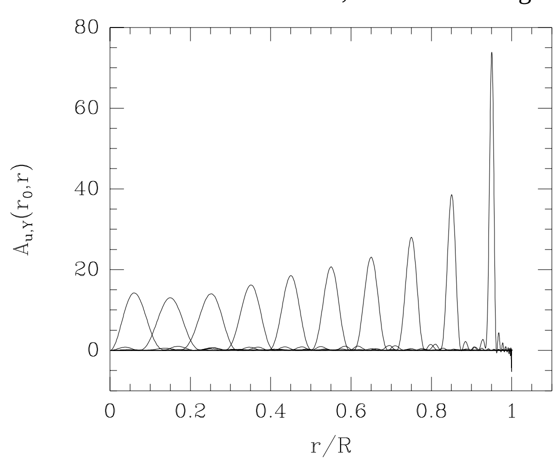

Figure 1: *Localized averaging kernels, $A_{u,Y}$, that are linear combinations of the corresponding seismic kernels $K_{u,Y}$*

If deviations of structure properties are represented in parametric form, then the unknown parameters can be evaluated from the helioseismic equations (1) using a least-squares technique. Kosovichev (1993) applied this calibration technique to determine the helium abundance and the depth of the convection zone. Finally, super-resolution techniques can be developed to study particular features of the interior structure, such as overshooting and sharp variations of the interior properties by applying nonlinear constraints, for instance.

The nonspherical axisymmetric perturbations can be considered in the parametric form:

$$\delta f(r,\theta) = \sum_{\lambda>0} \delta f_\lambda(r) P_\lambda(\cos\theta), \quad (3)$$

where P_λ are Legendre polynomials. In this case radial functions $f_\lambda(r)$ are determined by inverting so-called 'a-coefficients', expansions of the symmetrical part of split-multiplet frequencies into a set of orthogonal functions of azimuthal order m. The integral equations relating the even a-coefficients to $f_\lambda(r)$ are essentially identical to Eq. (1) (e.g. Gough, 1993). Therefore, a unified inversion procedure for determining spherical and aspherical components of the solar structure is being developed. Gough and Kosovichev (these Proceedings) have carried out the nonspherical structure inversion to determine latitudinal dependence of the depth of the convection zone.

An example of the 'primary' inversion for the spherically-symmetrical component of the solar structure is shown in Fig.2. The inverted data combine frequencies of 16 low-degree modes (l = 0, 1 and 2) in the frequency range $2.5 \lesssim \nu \lesssim 3$ mHz, taken from the IPHIR data set (Toutain & Fröhlich, 1992) with 598 frequencies of intermediate-degree modes (l = 4 – 140, ν = 1.5 – 3 mHz) observed at the BBSO by Libbrecht *et al.* (1990) in 1988, approximately at the same period of time when the IPHIR space experiment was carried out.

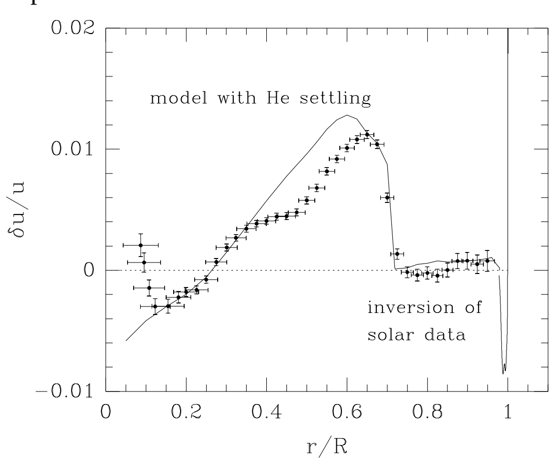

Figure 2: *Optimally localized averages of the difference $\delta u/u$, where $u \equiv p/\rho$, between the Sun and reference standard solar model 1 of Christensen-Dalsgaard et al. (1993). The horizontal bars represent the resolution lengths; the vertical bars represent standard errors. The continuous curve is the difference $\delta u/u$ between model 2 of Christensen-Dalsgaard et al., which accounts for helium settling against diffusion in the absence of turbulent mixing, and the reference.*

The inversions were compared with non-standard solar models of Christensen-Dalsgaard *et al.* (1993), in which gravitational settling of helium and turbulent diffusion have been taken into account. The model that assumes no mixing by turbulence (continuous curve) provides a good representation of the hydrostatic stratification of the interior outside the solar core. The structure of the energy-generating core remains uncertain because while the IPHIR data indicate a rise of $\delta u/u$ towards the center, which could be evidence of local material redistribution, similar data from the Birmingham Oscillation Network Group (BISON) (Elsworth, *et al.* 1994) yield no such trend (Gough & Kosovichev, 1993). However, in the both cases, the deviations in the core are too small to affect significantly the fluxes of solar neutrinos.

2. THE OPAL EQUATION OF STATE AND OPACITY EFFORT

by F. J. Rogers and C. A. Iglesias

The inability to resolve a number of long-standing discrepancies between theory and stellar observations led to the speculation that the widely used Los Alamos opacities were missing important sources of opacity (Simon 1982; Christensen-Dalsgaard *et al.* 1985). Due to this speculation and the need for the opacity of low Z materials to model laser produced plasmas, the OPAL opacity effort was undertaken (Iglesias, Rogers & Wilson 1987; 1992; Iglesias & Rogers 1991; Rogers & Iglesias 1992a). While the development of a detailed equation of state (EOS) was necessary to calculate the occupation numbers needed for opacity calculations, the publication of EOS tables for astrophysical use was deferred due to the relatively large tables required to calculate accurately derivatives of EOS quantities. Historically stellar models have been computed inconsistently, since the EOS used to compute the opacities is not used throughout the rest of the model. The success of the OPAL opacities in helping to improve theoretical models (e.g., Moskalik, Buchler, & Marom 1992; Guenther *et al.* 1992; Swenson *et al.* 1994) has made it essential to also provide EOS data that is consistent with the opacity data.

The OPAL equation of state is based on an activity expansion of the grand canonical partition function of the plasma (Rogers 1994) in terms of its fundamental constituents (electrons and nuclei). The formation of composite particles and the many-body effects on internal bound states occur naturally in this approach. Hence, pressure ionization is a natural consequence of the theory. In contrast commonly used approaches, all of which are based on minimization of the free energy, are forced to assert the effect of the plasma on composite particles and must rely on an ad hoc treatment of pressure ionization. Another advantage of the OPAL approach is that it gives a systematic expansion in the Coulomb coupling parameter that includes subtle quantum effects generally not considered in other EOS calculations.

Since the elements above Ne have little effect on the EOS and disproportionately increase the computer requirements, they are not included in the currently available EOS tables. These tables cover the temperature range 5×10^3 to 10^8 degrees for densities up to 10^5 g/cc. We have tabulated data for X=0.0, 0.2, 0.4, 0.6, 0.8 and Z=0.0, 0.02, 0.04. An auxiliary code is available for interpolation in the variables Temperature, density, X, and Z. Due to the extreme accuracy of the upcoming SOHO and GONG data, future tables will include the effects of elements up to Fe and also some additional derivatives required to analyse the data. For the opacity calculations it was apparent from the beginning that the most computationally intensive part of the opacity calculations would be the vast amount of atomic data needed for bound-bound and bound-free absorption cross- sections. For this purpose we developed a parametric potential method that is fast enough to allow on-line calculations, while achieving accuracy comparable to single configuration Dirac-Fock results (Rogers, Iglesias, and Wilson 1988; Iglesias, Rogers and Wilson 1992). This on-line capability also provides flexibility to study easily the effects of atomic physics approximations such as various angular momentum couplings or data averaging methods.

It is interesting to note that the EOS not only plays an important role in modeling the helioseismic data, but also indirectly through the opacity. It has been shown that uncertainties in the EOS in the solar radiative interior are a major contribution to opacity uncertainties (Iglesias & Rogers 1995 and references therein). We include degeneracy and plasma collective effects in the free-free absorption using a screened form of the parametric potentials. Similar corrections to the Thomson scattering are obtained from the method of Boercker (1987). The spectral line broadening for one, two, and three electrons ions are obtained from a suite of codes provided by R.W. Lee (1988) that include linear Stark theory. For all other transitions we use Voigt profiles where the Gaussian width is due to Doppler broadening and the Lorentz width is due to natural plus electron impact collision broadening (Dimitrievic and Konjevic 1980).

Rosseland mean opacity tables for PopI element abundances are available for values of hydrogen mass fraction $0 \leq X \leq 1$ and metal mass fraction $0 \leq Z \leq 0.1$. Opacity tables for carbon and oxygen rich mixtures (Iglesias and Rogers 1993) and for alpha enhanced element abundances are also available. Interpolation codes are provided to help facilitate use of this extensive data. Note that our equation of state is for a fully coupled multicomponent plasma, rather than compilations of pure element calculations later combined using ideal gas mixing laws. Consequently, the original calculations assumed the Anders-Grevesse (1989) photospheric solar metal abundance. However, the current tables have been updated and are for the Grevesse 1991 solar metal abundance. Note that new tables for different metal compositions (e.g. Swenson *et al.* 1994; Kovacs *et al.* 1992) can be readily computed on request from the existing OPAL monochromatic opacities using a corresponding states method (Rogers and Iglesias 1992b). Requests should be addressed to opal@ocfmail.ocf.llnl.gov. New tables that include the effects of seven additional heavy elements (P, Cl, K,Ti, Cr, Mn, Ni) not included in the existing tables are just becoming available. These tables are based on the Grevesse and Noels (1993) abundances and also include some physics updates.

3. A NEW SOLAR MODEL

by J. Christensen-Dalsgaard

The availability of extensive sets of precise observed solar oscillation frequencies allows detailed tests of models of the solar interior structure. It is evidently important to compute solar models which includes as accurate a representation of the known physics of the solar interior as possible. Only in this way may we test whether our current ideas about the physics and other properties of the Sun are correct, or whether modifications are required.

The recently obtained frequencies from the LOWL instrument (Tomczyk *et al.* 1995) provide for the first time an extensive set of data spanning modes from degree 0 to fairly high degree, allowing inversion to infer properties of the solar structure from the core to the base of the convection zone and slightly beyond. As discussed by Basu *et al.* (these Proceedings) the use of such a consistent set offers substantial advantages over the previously used sets based on combining results from different instruments and obtained at different epochs; in particular, inversion for the structure of the core appears to be quite sensitive to possible inconsistencies introduced in such data combinations.

To utilize fully the new data I have computed a new model of the present Sun, attempting to use the best physics and parameters available and ensuring adequate numerical precision. Relative to the reference model used by Basu *et al.* (these Proceedings) the improvements mainly concern the opacity interpolation and the parameters for the nuclear energy generation. The model used the MHD equation of state (e.g. Mihalas *et al.* 1988), and OPAL opacities (Iglesias *et al.* 1992) including spin-orbit correction and based on the most recent photospheric iron abundance, with low-temperature opacities obtained from Kurucz (1991); opacity interpolation was carried with a procedure kindly provided by G. Houdek and J. Rogl (see Houdek & Rogl 1993), using a nine-parameter interpolating function defined in piecewise fashion on a cubic curved network. Compared with the earlier model a revised set of nuclear parameters was used, from Bahcall & Pinsonneault (1995); also, the initial abundances of the CNO elements were revised, to use the values provided by Grevesse & Noels (1993). The calculation included diffusion and settling of helium, treated in the manner of Christensen-Dalsgaard, Proffitt & Thompson (1993); diffusion of heavier elements was neglected.

A detailed description of the calculation and the results, as well as of the differences between the Sun and the model as inferred from inversions, is given by Basu *et al.* (1995). The dominant change relative to the reference model of Basu *et al.* (these Proceedings) is an increase by about 0.002 in the hydrogen abundance required to calibrate the model to solar luminosity, resulting from the changes in the nuclear parameters. This leads to changes of generally up to about 1 per cent in density (with somewhat larger effects in the superficial layers) and a change of about 0.2 per cent in the sound speed in the solar core. The general effect is to bring the model into somewhat better agreement with the observations than the model considered by Basu *et al.*, although there remain significant differences, particularly in the density in the core where the inferred solar value is smaller than the model by about 0.8 per cent. Some possible causes for this discrepancy are discussed by Basu *et al.* (1995).

4. A SEISMIC SOLAR MODEL DEDUCED FROM THE SOUND SPEED DISTRIBUTION

by H. Shibahashi

Helioseismology has provided the direct information about the solar interior as the products of the helioseismic inversion of the frequency data,—for examples, the sound speed distribution $c(r)$. However, the physical quantities used usually in the stellar evolution theory are the density $\rho(r)$, the pressure $P(r)$, the temperature $T(r)$, the luminosity $L_r(r)$, and the mass $M_r(r)$ as functions of the distance from the stellar center r, and not the sound speed distribution $c(r)$. Hence it is desired to determine the solar interior structure from helioseismic data to be compared with the evolutionary models and to be compared with the detected neutrino flux (cf. Gough and Kosovichev 1990, Dziembowski *et al.* 1994, Antia and Chitre 1995, Kosovichev 1995b). In this paper, we report our attempt to produce a solar model based on the helioseismic data (Shibahashi 1992, 1995).

Among the assumptions adopted in constructing the standard solar model, the assumption of the hydrostatic equilibrium and the values of the present solar mass, radius, and luminosity should be definitely accepted. In comparison with these the other assumptions have less experimental support. Let us depart from the standard construction of a solar model and try to reconstruct a solar model by using only the experimentally well measured quantities. These quantities are the mass $M_\odot$, the radius $R_\odot$, the photon luminosity $L_\odot$, and the sound speed distribution $c(r)$ obtained from helioseismology. We should also assume that the sun is in hydrostatic equilibrium. Whether the sun is in thermal equilibrium is uncertain, since even if the real sun is not in thermal equilibrium it takes about 10^7 yrs for the sun to recover its equilibrium state. The justification of this assumption is made only by the solar neutrino flux measurement. In this paper, however, we assume the sun is in thermal equilibrium, since our present purpose is to construct a solar model, which is consistent with various observational data, with the changes of input physics from standard models as small as possible. In summary, our assumptions in reconstructing a solar model are as follows:

- the mass is $M_\odot$,
- the radius is $R_\odot$,
- the photon luminosity is $L_\odot$,
- the sound speed distribution $c(r)$ is that obtained from helioseismology,
- the model is in hydrostatic equilibrium, and
- the model is in thermal balance.

It should be noted that the difference between the present approach and the conventional programs for stellar structure is only the subject condition; —that is, the basic equations governing the stellar structure are solved with the given chemical composition distribution $X(r)$ in the conventional programs for stellar evolution, while they are solved with the given sound speed distribution $c(r)$ in the present approach. In our preliminary attempt, however, we assume a fully ionized perfect gas for the equation of state. The departure from this simple law is expected to be small in the bulk of the sun except near the photosphere, and it may be smaller than the uncertainties of the other physical quantities such as the opacity or the nuclear reaction rates. It should be noted that in this approximation the equations governing the hydrostatic balance are decoupled from those governing the thermal structure. That is, we can determine the pressure and the density distribution solely from the helioseismic observational data. More details of the present approach are given in Shibahashi *et al.* (1995).

We determine the density and the pressure distribution in the sun using the sound speed distribution, which was inverted by Vorontsov and Shibahashi (1991) from the observational data of the solar p-modes' frequencies obtained by Libbrecht *et al.* (1990) and Jiménez *et al.* (1988) and shown in Fig. 3. We also calculated the squared Brunt-Väisälä frequency N^2 from the density and the pressure profiles deduced from the sound speed profile, and we see that the core is convectively stable. The density profile thus obtained is shown in Fig. 4. The density thus determined is lower at the solar center than the standard models.

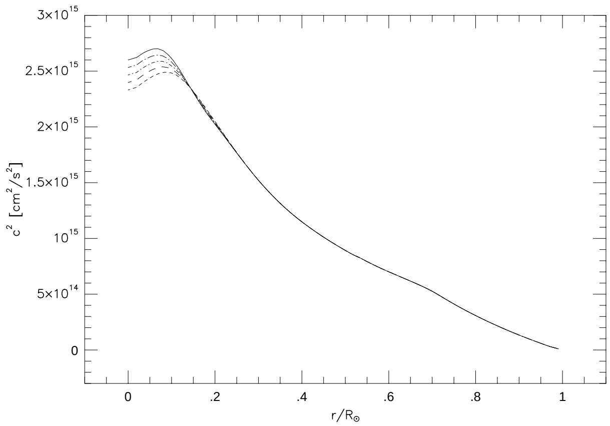

Figure 3: *The squared sound speed distribution inverted by Vorontsov and Shibahashi (1991) from the observed p-mode frequencies, which were obtained by Libbrecht et al. (1990) and Jiménez et al. (1988) and compiled by Libbrecht et al. (1990). Various curves are drawn within the observational error bars.*

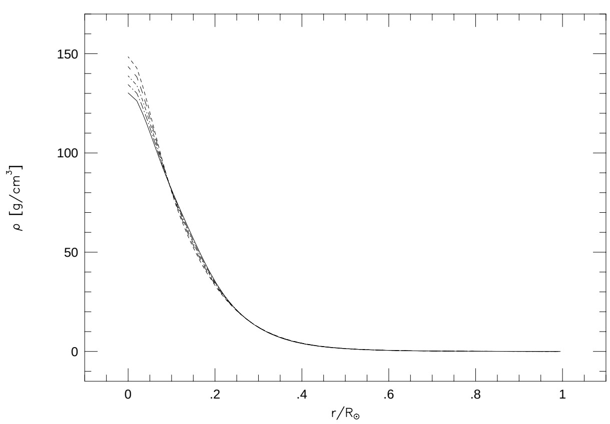

Figure 4: *The density profiles obtained from the sound speed profiles shown in Fig. 3.*

Following the conventional approach, we assume that Z is constant over the entire region of the solar interior and over the entire history of the sun; —that is, we assume that the abundance ratios of the various heavy elements in the solar interior are the same as those observed spectroscopically near the solar surface, —that is, the mass ratio of the heavy elements to hydrogen Z/X is 0.027 (Grevesse 1984, Aller 1986). Starting from the center and parameterizing the value of the central temperature, $T_{\rm c}$, we can integrate the equation governing the thermal structure of the sun with the given functional forms of $c^2(r)$, $\rho(r)$, and $P(r)$. and search for the parameter value satisfying the condition $L_r = L_\odot$ at $r = R_\odot$.

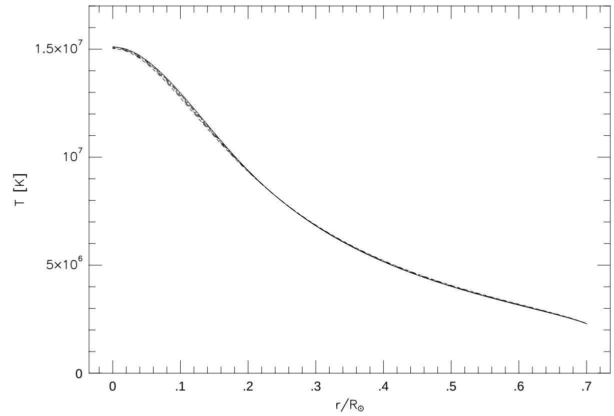

Figure 5: *The temperature profiles corresponding to the modified sound speed distributions shown in Fig. 3.*

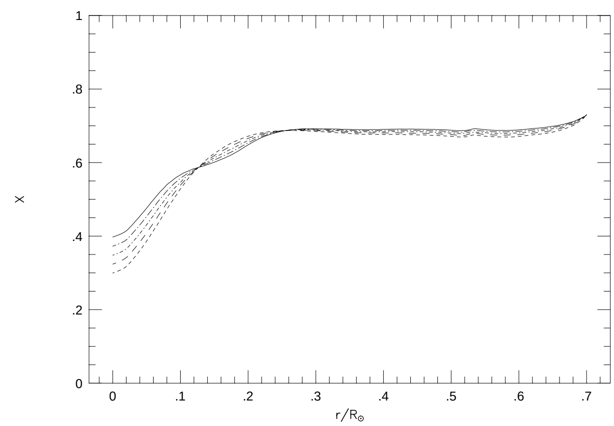

Figure 6: *The hydrogen abundance profiles X corresponding to the modified sound speed distributions shown in Fig. 3.*

Figures 5 and 6 show the temperature distribution and the hydrogen abundance distribution thus determined, respectively, corresponding to the sound speed distribution shown in Fig. 3 and the density profile shown in Fig. 4. Compared with these figures, we see that the difference in the sound speed structure does not give so strong influence on the temperature structure, but its effect on the chemical profile is substantial.

Once we get the thermal structure, we can check the accuracy of this approximation and can estimate a better equation of state and a better value for Γ_1. Then we can repeat the procedure to get more consistent solutions. By iterating in this way, we will get a reasonable solar model based on the experimentally reliable data.

5. INVERSION OF COMBINED AND HOMOGENEOUS DATA SETS

by Sarbani Basu

Observations of solar oscillations have provided us with accurate measurements of p-mode frequencies which severely constrain the structure of the Sun. Inversion of available solar frequencies has revealed that current solar models differ from the Sun by only a few percent in sound speed and density (e.g., Däppen *et al.*, 1991; Antia & Basu, 1994; Dziembowski *et al.*, 1994). However, these inversions suffer from one drawback — they were based on more than one set of data.

Generally the frequencies of low-degree ($0 \leq \ell \leq 3$) modes are obtained from one source and higher-degree modes from another. Such combinations of data may introduce systematic errors in the inversion results due to differing effects of magnetic activity in observations obtained at different epochs or because of different characteristics of the instrument.

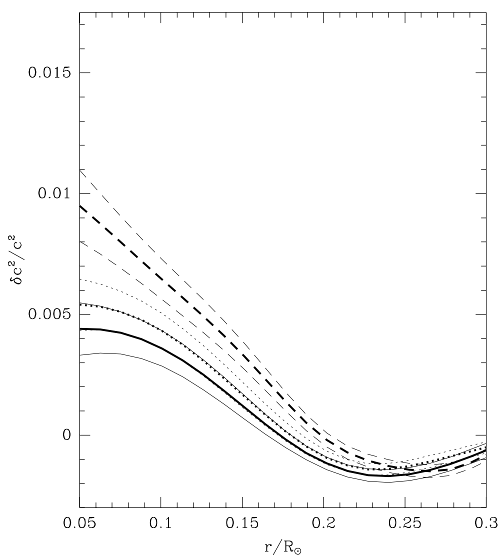

Figure 7: *The inversion results for different mode sets with low-degree data from the BISON network. The thick dashed line is for combination B1 (BISON & BBSO with frequency cut-off at 5 mHz), thick dotted line for combination B2 (BISON & BBSO with frequency cut-off at 3.5 mHz), and thick solid line are the results for combination B3 (BISON & LOWL). The thin lines show 1 σ error limits for each of the results.*

To check the effects of having mixed data we have considered two sets of frequencies of low-degree solar oscillations — the BISON (Elsworth *et al.*, 1994) and the IPHIR (Toutain & Fröhlich, 1992), and combined them with intermediate-degree data from BBSO (Libbrecht *et al.*, 1990). For both the low-degree sets, we make two combinations with the BBSO data, the first in which the intermediate-degree data has a frequency cut off at 5 mHz (we call this combination B1 and I1 for BISON and IPHIR respectively), and the second for which the frequency cut-off is 3.5 mHz (for convenience called combinations B2 and I2). The inversion results for the BISON-BBSO combination are shown in Fig. 7 and the IPHIR-BBSO combination in Fig. 8.

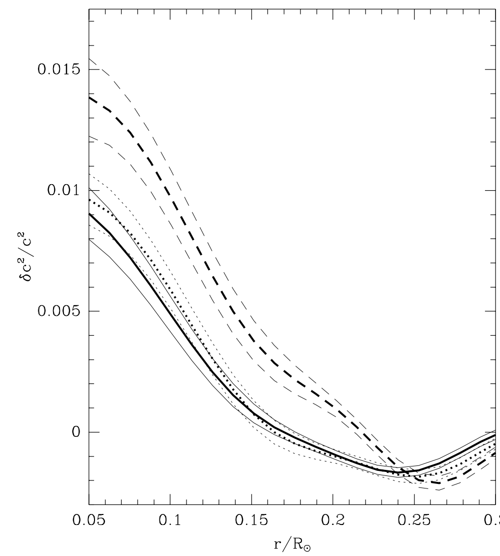

Figure 8: *The same as Fig. 8 but with the IPHIR low-degree data. Thus the thick dashed, dotted and solid lines are the results for sets I1, I2 and I3 respectively.*

The results are for the relative sound-speed difference between the Sun and a reference model. The details of the reference model and the inversion procedure can be found in Basu *et al.* (1995). The figures show the sound speed only for radii $r \leq 0.3$ R$_\odot$.

Since the low-*l* modes, which give information about the centre, are identical for B1 and B2, it could be expected that the inversion results shown in Fig. 7 should be similar. We would also expect the results for combinations I1 and I2 to be similar. However, we find that the results of B2 are more than 2σ lower than those of B1. Fig. 8 shows a similar behaviour for I1 and I2. This means that the uncertainties in the results in the core are greater than those indicated by the formal errors in the inversion. Since the same widths of the averaging kernels, as functions of target position, were used for the inversion of all sets of data, the effect of including high-frequency modes does not arise from an increase in resolution. Thus the difference is probably a result of systematic errors in the data. From Fig. (6) of Basu *et al.* we can see that there is very little change in the sound speed at larger radii.

To verify further that the inversion results at the core depends quite crucially on the intermediate-degree modes used, we combined the BISON and IPHIR frequencies with the intermediate-degree modes obtained with the LOWL instrument (see Tomczyk *et al.* 1995, Basu *et al.* 1995) to obtain mode-sets B3 and I3 respectively. The results of these sets are also shown in Figs. 7 and 8. We see that sound speed results obtained in the core for sets B3 and I3 are still lower by about 1σ compared with those of sets B2 and I2 respectively. This clearly indicates the importance of the higher-degree modes for inversions in the solar core.

Thus it is evident that great care is required when matching the very precise low-degree data that is available with higher-degree data for the purpose of carrying out inversions. Otherwise the uncertainties in the result in the solar core will not be truly represented by the error-bars of the solution. The effect of solar cycle changes may be cancelled by combining observations made at the same time. However, systematic effects due to differences in instrument response, methods of analysis etc., will remain. This emphasizes the value of carrying out inversion on homogeneous sets of data which have been obtained by the same instrument at the same time and analysed in the same way.

6. ON THE EFFECT OF CORRELATED ERRORS ON LINEAR INVERSIONS

by D.O. Gough and T. Sekii

It has been customary so far to formulate helioseismic inversions under the assumption that observational errors are independent and are Gaussian distributed, in spite of abundant evidence suggesting that that is not the case. This has been mainly due to the lack of more detailed information concerning statistics of errors supplied to inverse-problems solvers, and there has not been very much they can do about the situation, other than, for example, discarding outliers (perhaps erroneously).

To investigate the extent to which the simplified assumption on error statistics can affect inversions, we have studied the effect of correlated errors on linear inversions, by taking one-dimensional rotation inversion as a model problem. The frequency splittings were computed, and then correlated errors were added. Inversions were carried out first without, and subsequently with cognizance of the covariance matrix. In the former case only the diagonal elements of the covariance matrix were given and the off-diagonal elements were assumed to be zero.

In an extreme case, the correlated errors were generated by designing a covariance matrix in such a way that the correlation of errors conspires to present a systematic shift in frequency splittings. As was expected, inversions without taking account of the covariance matrix failed, and produced a systematic deviation from the true solution. On the other hand, using the proper covariance matrix led to a better result.

We have considered other cases, in which we have simulated a very simple data reduction procedure that generates correlated errors, and have computed the covariance matrix, under various assumptions. In the absence of a systematic shift in splittings (Fig. 9), the solutions do not depend dramatically on whether or not the correct covariance matrix was incorporated into the inversion algorithm. However, using an inappropriate covariance matrix led to larger error magnification (Fig. 10).

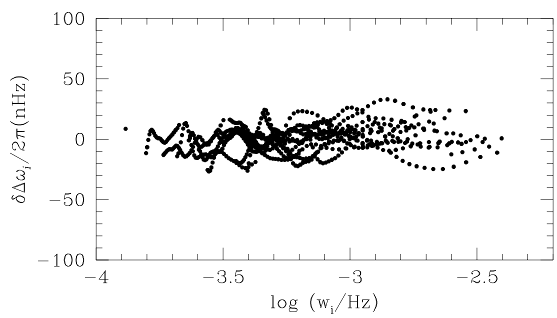

Figure 9: *An example of correlated errors plotted against $w_i \equiv \omega_i/(l+1/2)$, where ω_i and l are the frequency and the degree of the mode i. There is no correlation between modes of different radial orders.*

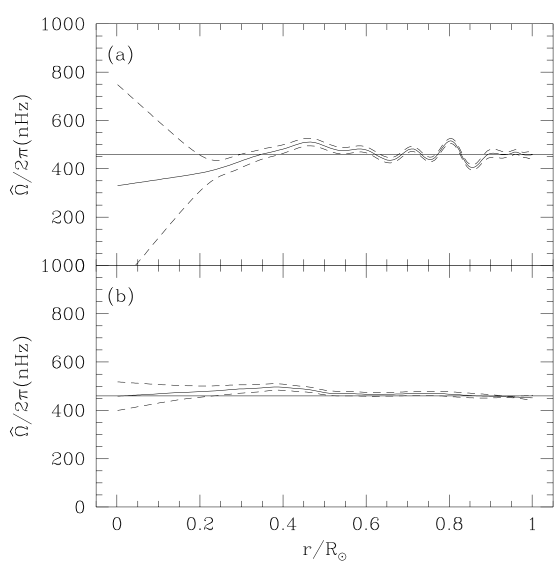

Figure 10: *Inversions of the rotational splittings for a rigid rotation of $\Omega/2\pi = 460$ nHz (horizontal line), with the errors of Fig. 9. (a)Obtained by ignoring the off-diagonal elements of the covariance matrix. (b)Obtained by using the correct covariance matrix. The dashed curves indicate 1σ level. The regularization parameters are adjusted to yield the same residual in the both cases.*

7. INVERSION OF THE SOLAR INTERIOR: PERSPECTIVES WITH GOLF & VIRGO DATA

by A. Eff-Darwich and F. Pérez-Hernández

A modified version of Regularized Least Squares (RLS) method has been used to invert helioseismic data. To test the inversion technique, the equilibrium differences between two theoretical models have been compared with the ones obtained from inversions. Mode set and errors are taken from actual observations. In order to see how GOLF & VIRGO data modify the results, low l, low frequency p-modes and g-modes have been added to present mode set, and the whole data set has been inverted. Results show an important improvement in the inversion, specially in the deeper part of the Sun.

One of the main problems in helioseismic inversions is that the matrix to be inverted is almost singular. Such singularity leads to a solution with great error magnification and oscillatory components. A modified version of SVD method has been developed in order to avoid such problems in the solution. SVD method decomposes a matrix A into three ones:

$$A = U \cdot \Lambda \cdot V^T \tag{4}$$

where Λ is a diagonal matrix that contains the eigenvalues of A. U and V are orthogonal matrixes where V contains the eigenvectors of A.

Each tabular point **a** has its own contribution to the diagonal matrix Λ. The contribution of each of the N tabular points is a matrix Λ_a:

$$\Lambda = \sum_{a=1}^{N} \Lambda_a \tag{5}$$

$$\Lambda_a = \Lambda \cdot W_a^T \cdot V \tag{6}$$

If w_a^{ij} is an element of matrix W_a and v^{ij} is an element of V then:

$$w_a^{ij} = v^{ij}\delta_{ia} \quad j = 1, N \tag{7}$$

The maximum value in Λ_a is used as a measure of the contribution of each tabular point **a** to the inversion. This information is used in order to build an optimal integration mesh (avoiding points where $max(\Lambda_a)$ are very low in comparison with the other ones). Because RLS technique is used in the inversion procedure, it is necessary an smoothing function weighted by an smoothing parameter. Such parameter is usually taken as a constant, but in the modification used here each tabular point **a** has an independent smoothing parameter γ_a, that has been taken proportional to $max(\Lambda_a)$:

$$\gamma_a \propto max(\Lambda_a) \tag{8}$$

To test the inversion technique, the differences between two theoretical models have been compared with the ones obtained from inversions. Figure 11 shows the result using p-modes with $0 \le l \le 100$. Because the number of high degree p-modes used is small, inversion tests are applied only to $r/R < .9$. Figure 12 shows the result when g-modes with $1 \le l \le 5$ and low frequency, low l ($0 \le l \le 4$) p-modes (those expected from GOLF & VIRGO experiments) are added to the mode set. The solution obtained for density is improved because g-modes are very sensitive to density variations and deeper points have an important weight in the inversion.

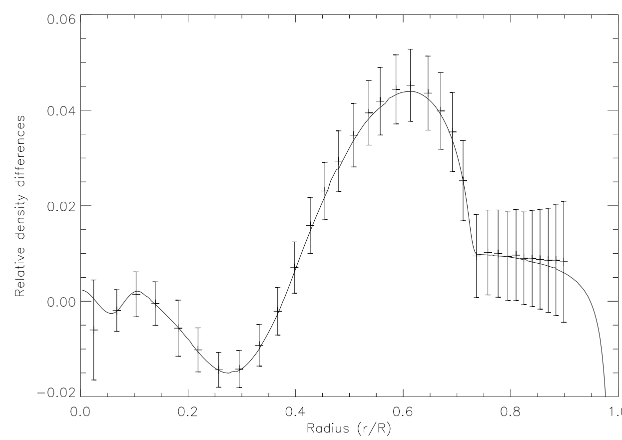

Figure 11: *Inverted results for density using p-modes with $0 \le l \le 100$. In solid line differences between the two models are shown.*

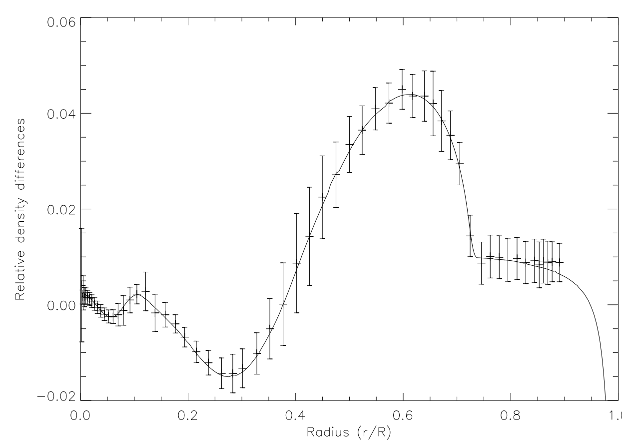

Figure 12: *Inverted results for density using p-modes with $0 \le l \le 100$, g-modes with $1 \le l \le 5$ and low frequency, low l ($0 \le l \le 4$) p-modes. In solid line differences between the two models are shown.*

REFERENCIES

1. Aller, L. H. 1986, in *Spectroscopy of Astrophysical Plasmas*, eds. A. Dalgarno and D. Layzer (Cambridge University Press), p. 89

2. Anders, E. & Grevesse, N. 1989, *Geochim. Cosmochim. Acta*, **53**, 197

3. Antia H. M., Basu S., 1994, *A&AS*, **107**, 421

4. Antia, H. M. and Chitre, S. M. 1995, *ApJ* in press

5. Backus, G. and Gilbert, F. 1968, *Geophys. J. R. astr. Soc.*, **16**, 169 - 205

6. Bahcall, J. N. & Pinsonneault, M. H., 1995. *Rev. Mod. Phys.*, in the press.

7. Basu, S., Christensen-Dalsgaard, J., Schou, J., Thompson, J. & Tomczyk, S., 1995. *Astrophys. J. Lett.*, submitted.

8. Boercker, D.B. 1987, *ApJ*, **316**, L95

9. Christensen-Dalsgaard, J., Duvall, T. L., Gough, D. O., Harvey, J. W. and Rhodes, E. J., 1985, *Nature* **315**, 378

10. Christensen-Dalsgaard, J., Proffitt, C. R. & Thompson, M. J., 1993. *Astrophys. J.*, **403**, L75 – L78.

11. Däppen W., Gough D. O., Kosovichev A. G., Thompson M. J., 1991, in: *Challenges to theories of the structure of moderate-mass stars, Lecture Notes in Physics*, vol. **388**, eds Gough, D. O., Toomre, J., Springer, Heidelberg, p. 111

12. Dimitrievic, M.S. & Konjevic, N 1980, *J.Quant.Spectrosc.Rad.Transf.*, **24**, 451

13. Dziembowski, W.A., Pamyatnykh, A.A.& Sienkiewicz, R., 1990, *Mon. Not. R. astr. Soc.*, **244**, 542-550.

14. Dziembowski, W. A., Goode, P. R., Pamyatnykh, A. A. and Sienkiewicz, R. 1994, *ApJ*, **432**, 417

15. Elsworth Y., Howe R., Isaak G.R., McLeod C.P., Miller B.A., New R., Speake, C. C., Wheeler S.J. 1994, *ApJ*, **434**, 801

16. Gough, D.O., 1993, in: *Astrophysical Fluid Dynamics*, eds. J.-P. Zahn and J. Zinn-Justin, Elsevier Science Publ. B.V.

17. Gough, D.O. & Kosovichev, A.G., 1988, in: *Oscillations of the Sun and Sun-like Stars*, ESA, 195-201.

18. Gough, D. O. and Kosovichev, A. G. 1990, in *Proc. of IAU Colloq. No. 121, Inside the Sun*, eds. G. Berthomieu and M. Cribier (Kluwer), p. 327

19. Gough, D.O. & Kosovichev, A.G., 1993, *Mon. Not. R. astr. Soc.*, **264**, 522-528.

20. Gough, D.O., Kosovichev, A.G. & Toutain, T., 1995, *Solar Physics*, **157**, 1-15

21. Grevesse, N. 1984, *Physica Scripta*, **T8**, 49

22. Grevesse, N. 1991 (private communication; see Iglesias, C.A., Rogers, F.J. and Wilson, B.G. 1992a for details)

23. Grevesse, N. and A. Noels, 1993, N. Prantzo, E.Vangioni-Flam, and M. Casse eds. in *Origin and Evolution of the Elements* (Cambridge Univ. Press, Cambridge).

24. Guenther, D. B., P. Demarque, Y.-C. Kim, and M. H. Pinsonneault, 1992, *ApJ*, **387**, 372.

25. Houdek, G. & Rogl, J., 1993. *Communications in Asteroseismology*, ed. Weiss, W. W., No. 60

26. Iglesias, C.A. & Rogers, F.J. 1991, *ApJ*, **371**, 408

27. Iglesias, C.A. & Rogers, F.J. 1993, *ApJ*, **412**, 752

28. Iglesias, C.A. & Rogers, F.J. 1995, *ApJ*, **443**,460

29. Iglesias, C.A., Rogers, F.J., & Wilson, B. G. 1987, *ApJ*, **322**, L45

30. Iglesias, C.A., Rogers, F.J., & Wilson, B. G. 1992, *ApJ*, **397**, 717

31. Jiménez, A., Pallé, P. L., Perez, J. C., Régulo, C., Roca Cortés, T., Isaak, G. R., McLeod, C. P. and van der Raay, H. B. 1988, in *Proc. of IAU Symp. No. 123, Advances in Helio- and Asteroseismology*, eds. J. Christensen-Dalsgaard and S. Frandsen (Reidel) p. 205

32. Kosovichev, A.G. & Perdang, J. 1988, in: *Oscillations of the Sun and Sun-like Stars*, ESA, 1988, 539-546.

33. Kosovichev, A.G. & Fedorova, A.V., 1991, *Sov. Astron.*, **35**, 507-14.

34. Kosovichev, A.G., 1992, Solar structure inversion package, SOI-MDI Technical Notes, SOI-TN-084, Stanford University.

35. Kosovichev, A.G., Christensen-Dalsgaard, J., Däppen, W., Dziembowski, W., Gough, D. & Thompson, M., 1992, *Mon. Not. R. astr. Soc.*, **259**, 536

36. Kosovichev, A.G, 1993, *Mon. Not. R. astr. Soc.*, **265**, 1053-1056.

37. Kosovichev, A.G., 1995a, *Advances in Space Research*, **15**, 95-99

38. Kosovichev, A. G. 1995b, in *GONG '94 : Helio- and Asteroseismology from the Earth and Space*, ed. R. K. Ulrich, E.J. Rhodes, W. Däppen (ASP Conference series), 89-100

39. Kovacs, G., Buchler, J.R., Marom, A., Iglesias, C.A., and Rogers, F.J. 1992, *A&A*, **259**, L46

40. Kurucz, R. L., 1991. In *Stellar atmospheres: beyond classical models*, p. 441 – 448, eds Crivellari, L., Hubeny, I. & Hummer, D. G., NATO ASI Series, Kluwer, Dordrecht

41. Lee, R.W. 1988, *J.Quant.Spectrosc.Rad.Transf.*, **40**, 561

42. Libbrecht, K. G., Woodard, M. F. and Kaufman, J. M. 1990, ApJS 74, 1129

43. Mihalas, D., Hummer, D. G., Mihalas, B. W. & Däppen, W., 1990. *Astrophys. J.*, **350**, 300 – 308.

44. Moskalik, P., J. R. Buchler, and A. Marom, 1992, *ApJ*, **385**, 685.

45. Rogers, F.J. & Iglesias, C.A. 1992a, *ApJS*, **79**, 507

46. Rogers, F.J. & Iglesias, C.A. 1992b, *ApJ*, **401**,361

47. Rogers, F.J. 1981, *Phys. Rev.*, 1994, in *The Equation of state in Astrophysics*, IAU Colloquiun 147, edited by G. Chabrier and E. Schatzman (Cambridge University Press, 1994)

48. Shibahashi, H. 1992, in *Frontiers of Neutrino Astrophysics*, eds. Y. Suzuki and K. Nakamura (Universal Academy Press), p. 93

49. Shibahashi, H. 1995, in *Condensed Matter and Plasmas in Physics and Astronomy*, eds. S. Ichimaru and S. Ogata (Addison-Wesley Pub. Co.), p. 203

50. Shibahashi, H., Takata, M., and Tanuma, S. 1995, in *Proc. of the 4th Workshop of SOHO: Helioseismology*, ed. B. Battrick (ESA Publication Division, Noordwijk), in press

51. Simon, N.R. 1982, *ApJ*, **260**, L87

52. Swenson, F.J., Faulkner, J., Iglesias, C.A., Rogers, F.J., and Alexander, D.R. 1994,*ApJ*, **422**, L79

53. Tomczyk, S., Streander, K., Card, G., Elmore, D., Hull, H. & Caccani, A., 1995. *Solar Phys.*, in press.

54. Toutain T., Fröhlich C. 1992, *A&A*, **257**, 287

55. Vorontsov, S. S. and Shibahashi, H. 1991, *PASJ* **43**, 739