

Study Of Solar High-Frequency Modes Near The Acoustic Cut-off Frequency

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Abstract. The acoustic cut-off frequency ν_{ac} is related to the height of the potential barrier near the solar surface. Acoustic waves with frequencies greater than ν_{ac} are only partially reflected by the barrier. In this paper we use a quantum mechanical potential well to model the acoustic potential ν_+ in the Sun. Using this simple model we are able to reproduce the shift in the frequency difference $\nu_{n,l} - \nu_{n-1,l}$ of acoustic p-modes around the acoustic cut-off frequency ν_{ac} as observed by Ronan (1994). This difference in frequency may be due to transmission resonances observed in atomic physics in the case of the Ramsauer effect.

Key words: Solar oscillation, acoustic cut-off frequency, Ramsauer effect

1. Introduction

The upper reflection layer is of particular interest in helioseismology because solar oscillation frequencies are very sensitive to the structure of this layer (Kumar et al. 1994). The variations are particularly large near the acoustic cut-off frequency, ν_{ac} , thus providing potential for probing structural changes in the subphotospheric layer. However, the physics of the observed frequency shifts near ν_{ac} have not been understood. A plot of the frequency difference ($\nu_{n,l} - \nu_{n-1,l}$) in micro Hz against frequency $\nu_{n,l}$ in micro Hz for $l=21$, $n=15$ to 35 from the South Pole data (Duvall, 1991) is shown in Fig. 1a. This plot suggests a dip near the acoustic cut-off frequency ν_{ac} , which is also seen in our theoretical model (Fig. 1b).

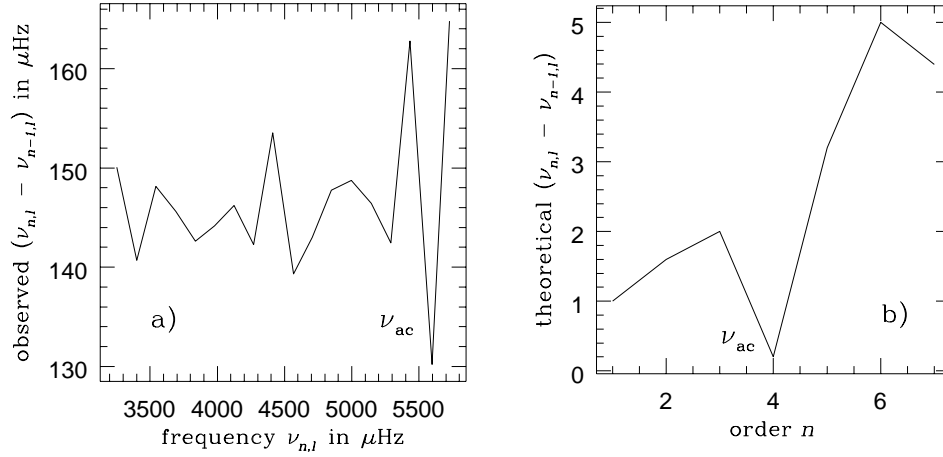


Figure 1: a) Frequency Difference Against Frequency From the South Pole Data for $l = 21$, $n = 15$ to 35. b) Computed Frequency Difference in Arbitrary Units vs Order for Rectangular Potential Well.

2. Problem Formulation

In this paper we solve the reduced Wave Equation (1) for a potential well. An infinite potential is taken at one end of the well and a finite potential at the other end (See Fig. 2b). The sound speed c_S is assumed to be constant as we have ignored the effects of stratification. The Sommerfeld Radiation condition (4) is applied far away from the finite barrier. This ensures outgoing waves. The equation is solved numerically using finite differences. The numerical solution is found to be in good agreement with the existing theoretical solutions. As a result of this there are two kinds of solutions for which the eigenfrequencies are quantized. (i) Bound States for which the frequencies are real and (ii) Scattered States for which the frequencies are complex, which is due to the fact that the differential operator becomes Non-Hermitian .

$$V'' + k_r^2 V = 0 \quad (1)$$

where V is the velocity perturbation which is a function of one spatial variable r , prime denotes differentiation with respect to that spatial variable, k_r is the vertical component of the local wave number and is given by

$$k_r^2 = \frac{(2\pi\nu)^2}{c_S^2} \left(1 - \frac{\nu_+^2}{\nu^2}\right) \left(1 - \frac{\nu_-^2}{\nu^2}\right) \quad (2)$$

c_S is the sound speed, ν is the wave frequency.

The parameter ν_+ plays the role of the acoustic potential for solar acoustic oscillations (See Fig. 2a), and ν_- is the potential for gravity modes. Since the frequencies of the p-modes are much higher than ν_- ,

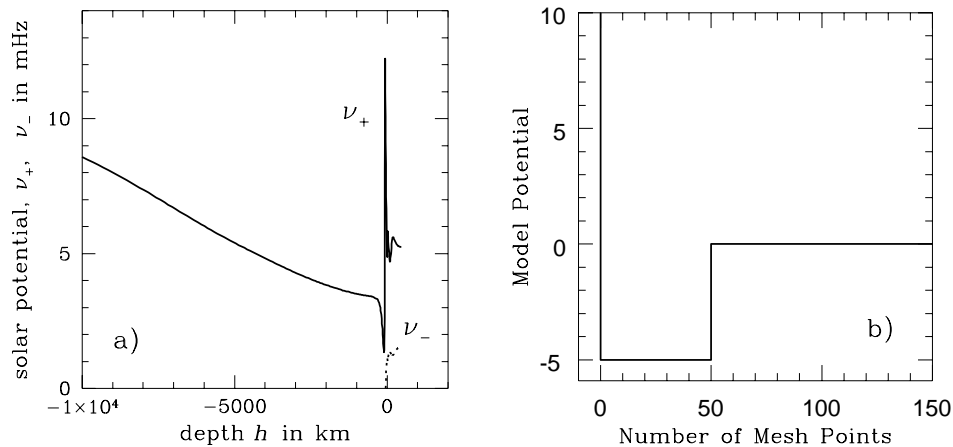


Figure 2: a) Solar Acoustic Potential. b) Rectangular Potential Well in Arbitrary Units Approximating the Solar Potential.

$$k_r^2 \approx \frac{(2\pi\nu)^2}{c_S^2} \left(1 - \frac{\nu_{\pm}^2}{\nu^2} \right) \quad (3)$$

The solar acoustic potential has a strong peak just below the photosphere (because of a strong density gradient) and tends to a constant value of 5 mHz in the chromosphere.

The Sommerfeld radiation condition

$$V' - ik_{r,0}V = 0 \quad (4)$$

is applied far away from the finite potential well, where $k_{r,0}$ is the value of k_r at the point of application of the radiation condition.

The lower boundary condition $V = 0$ is applied at $r = 0$.

3. Results and Conclusions

The eigenfunctions of the bound states oscillate inside the well and decay outside. The corresponding eigenfrequencies are real and are quantized due to the matching conditions at the boundary of the finite potential. The eigenfunctions of the scattered states oscillate inside the well and are matched to the propagating outgoing damped solutions as a result of the radiation condition. This results in the eigenfrequencies being quantized and complex. From numerical experiments it was determined that roughly one-third of the frequencies are accurate as they closely matched the analytical results.

In Fig. 1b the computed frequency difference $\nu_{n,l} - \nu_{n-1,l}$ in arbitrary units is plotted against the order n . In this plot a significant dip is seen around the acoustic cut-off

frequency ν_{ac} , thereby reproducing the observational result of Ronan (1994). The cause for the dip may be due to transmission resonances which are observed in atomic physics in the case of the Ramsauer effect. The waves suffer phase differences at the well edge as a result of which their frequencies change around the acoustic cut-off frequency ν_{ac} . The transmission and the frequency change depends on the amount by which the phase changes at the well edge.

4. Directions For Future Work

The next step is to apply this simple model to the real solar potential ν_+ and incorporate stratification effects. Varying the width of the potential amounts to varying the degree l . Various non-reflecting boundary conditions will be formulated at the upper boundary which would avoid spurious numerical reflections. A stochastic source term will be added to the right hand side of equation (1) and the resulting non-stationary spectrum will be analysed by the wavelet transform (Milford et al. 1994). To account for variations with the solar cycle, magnetic effects would be incorporated into the solar model. Maps of phase shifts due to the inhomogeneous medium in the presence of magnetic field will be calculated from an asymptotic analysis of equation (1) for high ν , and also from time-distance helioseismology (Jefferies et al. 1994).

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