SEISMIC OBSERVATION OF SOLAR TACHOCLINE

A.G. Kosovichev

W.W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305, USA

ABSTRACT

Helioseismology has established the existence of a layer of strong gradients of angular velocity at the base of the solar convection zone. This layer ('tachocline') separates the convection zone exhibiting a strong latitudinal differential rotation from the radiative interior that rotates almost rigidly. Turbulence generated in the tachocline is likely to mix material in the upper radiative zone resulting in the observed deficit of Li and Be. However, the theoretical estimates of the precise location and the thickness of the tachocline depends on the details of turbulent energy and momentum transport, and are uncertain (Spiegel & Zahn, 1992). The tachocline layer is also the most likely place for the solar dynamo. Using rotational splitting coefficients of the p-mode frequencies, obtained during 1986-90 at the BBSO, we have found that the thickness of the transitional layer is about 0.09 ± 0.04 solar radii, and that most of the transition occurs beneath the adiabatically stratified part of the convection zone, as suggested by the dynamo theories of the 22-year solar cycle.

1. Introduction

The transition layer at the base of the convection zone is considered as the most likely place for the solar dynamo. Within this layer, the toroidal magnetic flux which appears at the surface in various forms of the solar activity is generated from the radial component of poloidal field. The toroidal flux is believed to be mainly accumulated in a thin layer just beneath the convection zone because convection would quickly destroy the toroidal flux if the layer were widely extended into the convection zone.

Estimates of the thickness and precise location of the transition layer by standard helioseismic inversion techniques from rotational splitting of oscillation p modes are rather uncertain. Attempts to resolve the layer under global smoothness constraints lead to either an oversmoothed angular velocity profile or to spurious oscillations around the layer (cf Goode *et al.*, 1991). To overcome these difficulties, Goode *et al.* (1991) assumed that the transition layer is, in fact, discontinuous, and found that their model fits the helioseismic data best when the discontinuity coincides with the base of the convection zone. In this paper, we demonstrate that the discontinuous model of solar rotation is not the best fit to the data, and that a model with a transitional layer of finite thickness ($\approx 0.09 \pm 0.04R$) fits the data more accurately than the discontinuous model.

2. Frequency Splitting and Solar Rotation Law

Observed p-mode rotational frequency splitting is traditionally represented in the form (Duvall, *et al.*, 1986)

$$\Delta\nu_{nlm} = L \sum_{k=0,1,2,\ldots} a_{2k+1}(n,l) P_{2k+1}\left(\frac{m}{L}\right), \qquad (1)$$

where n, l, and m are the radial order, the angular degree, and the angular order of a normal mode respectively; $L = \sqrt{l(l+1)}$; P_k are Legendre polynomials. Thus, the observational data are sets of the 'odd a-coefficients', a_{2k+1}, for mode multiplets (n,l).

For the modes of intermediate and high degree, l, which probe the convection zone, the a-coefficients are directly related to the radial function $A_k(r)$ of the solar rotation law represented in terms of associated Legendre functions of order 1, $P^1_k(\theta)$ (Kosovichev, 1988; 1996)

$$\Omega(r,\theta)/2\pi = \sum_{k=0,1,2,\ldots} \alpha_k A_{2k+1}(r) \frac{P^1_{2k+1}(\theta)}{\sin\theta}, \qquad (2)$$

where $\alpha_k = (-1)^k \frac{k! 2^k}{(2k-1)!!}$. The contribution resulting from the Coriolis effect, does not exceed a few percent, and, therefore, can be treated as a small perturbation. Then, $a_k(n,l) \equiv a'_k(n,l)(1 - C_{nl})$, where C_{nl} is the Ledoux constant.

3. Estimates of Width and Location of the Transition Layer

We have used the splitting a-coefficients (a_1, a_3 and a_5) of approximately 800 multiplets of $l = 5-60$ obtained by Woodard and Libbrecht (1993) during 1986, and 1988-90 at the Big Bear Solar Observatory. Another available BBSO data set includes $a_1 - a_{11}$ coefficients of modes $l = 20 - 140$, however, it is less accurate than the first dataset. This dataset was used to estimate angular velocity near the surface. The a-coefficients corrected for the Coriolis effect, $a'_k(n,l)$, for each year have been grouped and weighted average computed in 16 equal intervals of r_t, between $0.5R$ and $0.9R$, each being $0.025R$ wide. Then, the weighted averages, $< a'_{k,j} >_{\rm obs}$ ($j = 1, \ldots, 16$), for the four years have been computed. The variation at the base of the convection zone is particularly strong for a'_3. Therefore, we have used this coefficient to estimate the thickness and the location of the transition layer.

Helioseismic inversions have revealed that the radiative interior essentially rotates rigidly. That means that, in rotation law (2), functions $A_k(r)$ for $k > 1$ are approximately equal to zero in the radiative zone. Also, the averaged $a'_3(n,l)$-coefficients suggest that A_3 is almost constant in the convection zone. Therefore, we have considered A_3 in the parametric form

$$A_3(r) = \frac{A_{3,0}}{2}\left(1 + \mathrm{erf}\left[\frac{2(r-r_0)}{w}\right]\right), \qquad (3)$$

where r_0 is the radius of the central point of the transition zone; and w is the characteristic thickness, erf is the error function; $A_{3,0} = 22$ nHz is determined from the flat part of a'_3 in the convection zone. Then, we have computed $< a'_3(n,l) >_{\rm mod}$ coefficients using Eq.(3) and the same averaging procedure that was applied to the BBSO data, and, finally, determined the mean squared

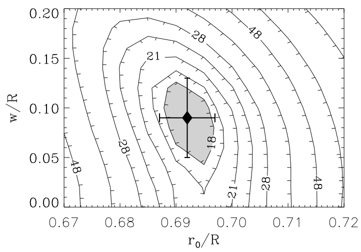

Figure 1: Contours of $\chi^2(w, r_0)$ evaluated from equation (7) at $\chi^2 = 18, 19, 21, 24, 28, 36, 48, 64$ and 96; w is the thickness of the transition layer, r_0 is its central radius. The shaded area corresponds to the increase of χ^2 by 1 from its minimum value, or 1σ uncertainty in the parameters. The error bars show the 1σ uncertainty estimated from statistical modeling by adding Gaussian noise to the data.

difference, $\chi^2(w, r_0)$. Function $\chi^2(w, r_0)$ shown in Fig. 1 reaches a minimum of 16.8 at $r_0 \approx 0.690 - 0.695R$ and $w \approx 0.08 - 0.10R$.

4. Seismic Model of Solar Rotation and Angular Momentum Transport

From the analysis of the BBSO data, we formulate a simple model of solar rotation based on the first three terms of expansion (2):

$$\Omega(x,\theta)/2\pi = A_1(x) + A_3(x)[1 - 5\cos^2\theta] + A_5(x)[1 - 14\cos^2\theta + 21\cos^4\theta], \quad (4)$$

where $x = r/R$ and

$$A_1(x) = \begin{cases} 435, & x \le 0.71 \\ 435 + 51.85(x - 0.71), & 0.71 < x \le 0.983 \\ 434 - 882.53(x - 1), & 0.983 < x \le 1 \end{cases}$$

$$A_3(x) = -22\,\Phi(x), \qquad A_5(r) = -3.5\,\Phi(x). \quad (5)$$

Here, $\Phi(x) = 0.5(1 + \text{erf}[2(x - 0.69)/0.1])$; the values of A_k are given in nHz. To approximate A_k at $r > 0.9R$, we have used the extended BBSO data set ($l = 20 - 140$) and the results of Doppler measurements of surface solar rotation. The model is illustrated in Fig.2. It is

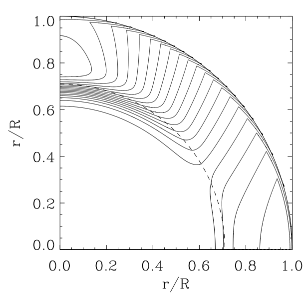

Figure 2: Contours of $\Omega(r, \theta)/2\pi$ of solar rotation model (8)-(9), from 328 nHz to 461 nHz separated by 7 nHz. The dashed circle shows the base of the convection zone.

interesting that the radial behavior of A_1 is very different from the variation of A_3 and A_5. Unlike these functions, A_1 does not show any sharp transition at the base of the solar convection zone. However, A_1 sharply decreases near the surface while A_3 and A_5 show no significant variation. This distinct behavior is probably related to the mechanisms of transport of angular momentum in the convection zone, because $A_1(r)$ represents the net angular momentum of a spherical shell of mass $dm = 4\pi\rho r^2 dr$: $dL = \frac{1}{3}A_1 r^2 dm$ (cf Goode and Dziembowski, 1993). Thus, the absence of sharp variation of A_1 at the base of the convection zone means zero net torque between the radiative and convection zones (Gough, 1985), and predominantly meridional angular momentum transfer in the transition layer. The gradual growth of A_1 in the bulk of the convection zone, approximated by a linear function in our model (5), can result from angular momentum transport towards the surface.

The sudden drop of A_1 near the surface is likely to be the result of the conservation of the angular momentum ($A_1 \propto 1/r^2$) maintained by strong radial convective flows in the supergranulation layer, as suggested by Foukal and Jokipii (1975). The depth of this subsurface layer has not yet been determined by helioseismology because that requires reliable measurements of oscillation modes of $l > 140$, which have the lower turning points at $r_t > 0.96R$. Therefore, we have employed the Foukal-Jokipii hypothesis to estimates the depth of this transition layer linearly interpolating between A_1 estimated seismologically at the top of the convection zone and A_1 measured on the surface (Snodgrass, 1992). From $A_1 r^2 = \text{const}$, $\Delta r = -r\Delta A_1/2A_1 \approx 0.017R$. It is interesting that the depth of this layer coincides with the depth of the HeII ionization zone, thus supporting the Simon-Leighton (1964) idea that supergranulation is driven by the ionization of helium.

5. Discussion

Estimated from the 1986-90 BBSO solar oscillation data, the thickness ($0.09 \pm 0.04R$) and the central position ($0.692 \pm 0.005R$) of the transition layer at the base of the solar convection zone are consistent with the requirements of the $\alpha\omega$-dynamo theory of the solar cycle. Another layer of strong angular velocity gradient is likely to be just beneath the solar surface. Matching the subsurface angular velocity from helioseismology with the direct Doppler measurements of the surface angular velocity, and assuming the conservation of the angular momentum, we have estimated that the layer is only 12 Mm ($0.017R$) deep. Evidence for a subsurface angular-velocity gradient has been seen in many helioseismic data sets. However, the steepness of the gradient is not established yet.

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