DETERMINATION OF SOLAR SUBSURFACE ROTATION BY TIME-DISTANCE HELIOSEISMOLOGY

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ABSTRACT

Differences of reciprocal acoustic travel times obtained from MDI data are used to infer the azimuthally averaged velocity of subsurface flows. The method essentially provides snapshots of the Sun's internal rotation. The initial results demonstrate variations of the angular velocity with depth and latitude, and reveal the North-South asymmetry of the internal rotation.

1. Introduction

Observations of rotation of large-scale magnetic structures on the solar surface show that the angular velocities of the north and south hemispheres may differ by about 5%. The asymmetry varies with time and correlates with the solar cycle providing important information about the mechanism of the solar cycle (Antonucci, Hoeksema, and Scherrer, 1990).

The traditional helioseismic techniques based on inversion of mode frequencies do not provide an effective measure of the asymmetrical component of the differential rotation because this component appears only as a second-order correction to the oscillation frequencies. In this measurement, the antisymmetric component has to be separated from the second-order terms of symmetric rotation and from contributions of non-spherical structure perturbations. The measurements are likely to be restricted to p modes of high degree ($l > 300$), implying that the asymmetry could be studied perhaps only in the subsurface layers (Gough and Kosovichev, 1995)

We explore how solar differential rotation can be measured by time-distance helioseismology (Duvall *et al.*, 1993). This technique provides local measurements of azimuthal flows, and, thus, has no restriction on the symmetry of the solar rotation.

2. Time-Distance Measurement

The basic idea of time-distance helioseismology is to measure the acoustic travel time between different points on the solar surface, and then to use the measurements for inferring variations of the structure and flow velocities in the interior along the wave paths connecting the surface points.

To measure the wave travel time, we use the cross-covariance function, $\Psi(\tau,\Delta)$, of the oscillation signal, $f(t,\boldsymbol{r})$, between different points on the solar surface (Duvall, *et al.*, 1993):

$$\Psi(\tau,\Delta) = \int_0^T f(t,\boldsymbol{r}_1) f^*(t+\tau,\boldsymbol{r}_2)dt, \tag{1}$$

where Δ is the angular distance between the points with coordinates $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, τ is the delay time, and T is the total time of the observations. Because of the stochastic nature of excitation of the oscillations, function Ψ must be averaged over some areas on the solar surface to achieve a signal-to-noise ratio sufficient for measuring travel times τ. The oscillation signal, $f(t,\boldsymbol{r})$, is usually the Doppler velocity or intensity.

We measure both the phase ($\tau_{\rm ph}$) and group ($\tau_{\rm g}$) travel times for a given angular distance, Δ, by fitting to the cross-covariance function a Gabor-type wavelet:

$$G(\tau,\Delta) = A\cos\left[\omega_0\{\tau-\tau_{\rm ph}(\Delta)\}\right]\exp\left[-\delta\omega^2\{\tau-\tau_{\rm g}(\Delta)\}^2\right], \tag{2}$$

where A is the amplitude, ω_0 is the central frequency of the frequency filter applied to the oscillation data, and $\delta\omega$ is the characteristic width of the filter.

Typically, we measure times for acoustic waves to travel between points on the solar surface and surrounding quadrants symmetrical relative to the North, South, East and West directions. In each quadrant, the travel times are averaged over narrow ranges of travel distance Δ to improve the signal-to-noise ratio. Then, the times for westward-directed waves are subtracted from the times for eastward-directed waves to yield the time, $\tau_{\rm diff}$, which predominantly measures West-East motions.

3. Time-Distance Calibration

We use the geometrical acoustic (ray) approximation to relate the measured phase times to the internal properties of the Sun. More precisely, we measure the variations of the local travel times at different points on the surface, relative to the travel times averaged over the observed area, and then infer variations of the internal structure and flow velocities from the travel time anomalies using a perturbation theory.

In the ray approximation, the travel times are sensitive only to the perturbations along the ray paths. The variations of the travel time obey Fermat's Principle

$$\delta\tau = \frac{1}{\omega}\int_\Gamma \delta\boldsymbol{k}d\boldsymbol{r}, \tag{3}$$

where $\delta\boldsymbol{k}$ is the perturbation of the wave vector due to the structural inhomogeneities and flows along the unperturbed ray path, Γ.

We separate the effects of flows from structural perturbations by taking the difference of the reciprocal travel times:

$$\delta\tau_{\rm diff} = -2\int_\Gamma \frac{(\boldsymbol{nU})}{c^2}ds, \tag{4}$$

where $\boldsymbol{U}$ is the flow velocity, c is the sound speed, $\boldsymbol{n}$ is a unit vector tangent to the ray.

If we assume that the velocity, $\boldsymbol{U}$, does not change significantly within a local ray system, then

$$\delta\tau_{\rm diff} = \frac{4\alpha U_x}{c_t}\int_0^{z_t}\frac{dz}{c^2\sqrt{c^{-2}-c_t^{-2}}} = \frac{2\alpha U_x}{c_t}\tau, \qquad (5)$$

where U_x is the East-West component of the flow velocity, z is the depth, z_t is the depth of the ray turning point, $c_t = c(z_t)$, $\alpha = 2\sqrt{2}/\pi$, τ is the travel time to the angular distance Δ. Then, using Benndorf's relation (Ben-Menahem, 1981), $c_t = R\left(\frac{d\Delta}{d\tau}\right)$, where R is the solar radius, we determine the velocity as a function of the travel time perturbation, $\delta\tau_{\rm diff}$, (Duvall, 1995):

$$U_x = \frac{R}{2\alpha}\left(\frac{1}{\tau}\frac{d\Delta}{d\tau}\right)\delta\tau_{\rm diff}. \qquad (6)$$

The coefficient of this equation can be measured by differentiating the observed time-distance relation. For a polytropic model of constant index μ

$$\tau = \sqrt{\frac{4\pi(\mu+1)R}{\gamma g}\Delta}, \qquad (7)$$

where γ is the adiabatic exponent, g is the gravity acceleration, thus,

$$U_x = \frac{\gamma g}{4\pi\alpha(\mu+1)}\delta\tau_{\rm diff}. \qquad (8)$$

Finally, the solar rotation rate in μrad/sec is

$$\Omega \approx \Omega_0 + 1.15\,\delta\tau_{\rm diff}\ \sec\lambda, \qquad (9)$$

where Ω_0 is the Carrington rotation rate ($\approx$ 2.666 μrad/sec), λ is latitude, and $\delta\tau_{\rm diff}$ is measured in minutes.

Figure 1 shows the solar rotation rate inferred by calibrating the variations of reciprocal travel time difference, $\delta\tau_{\rm diff}$ obtained from the MDI data. This rotation rate is an average over the subsurface layer approximately 10 Mm deep. These measurements cover only an 8-hour period; thus, the errors of the estimate are large. However, there is an indication that the rotation rate was systematically lower in the Southern hemisphere between -30 and -20 degrees.

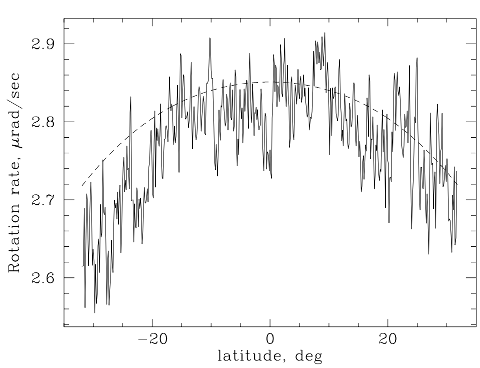

Figure 1: The solar rotation rate averaged over the subsurface layer approximately 10 Mm deep, as inferred from MDI data (solid curve), and the surface rotation rate (dashed curve).

4. Time-Distance Inversion

The solar rotation rate as a function of the depth and latitude has been inferred by inverting the integral equation (4) using a regularized least-squares technique (Kosovichev, 1996). The result is shown in Fig.2. The small-scale variations of the rotation rate are probably not significant. However, the slower rotation of the Southern hemisphere is evident. The variation of the rotation rate with depth is not very evident from the short time series we have analyzed.

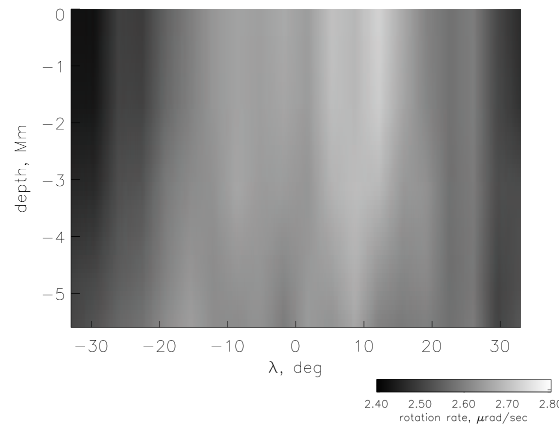

Figure 2: The solar rotation rate as a function of the depth and latitude inferred by inverting the MDI time-distance data.

5. Conclusion

We have demonstrated that time-distance helioseismology can provide unique information about the asymmetrical component of solar rotation. With this new technique, we can possibly detect local variations of solar rotation on a relatively short time-scale. The results presented in this poster were obtained from a 8-hour time series of MDI Doppler images. These initial results give evidence of currently slower rotation of the Southern hemisphere. Averaging for a longer period will substantially improve the accuracy of the measurements.

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