AN ATTEMPT TO MEASURE LATITUDINAL VARIATION OF THE DEPTH OF THE CONVECTION ZONE

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ABSTRACT

The location of the base of the convection zone coincides with the sharp variation of the slope of the relative difference in the quantity $u = p/\rho$ between the Sun and a solar model, provided that the zone of adiabatic convection is deeper in the model than in the Sun. We have determined the difference $\delta u/u$ as a function of radius at various latitudes between the Sun and a spherically symmetrical solar model by inverting the BBSO data (Libbrecht & Woodard, 1993). The results offer evidence that the convection zone may be somewhat deeper at the equator than it is at the poles. The variation of the depth, however, does not exceed $0.02 R_\odot$.

Keywords: *Sun's interior, oscillations, convection zone, tachocline, solar cycle, standard solar model*

1. INTRODUCTION

The tachocline (shear layer) at the base of the convection zone drives meridional circulation (e.g. Spiegel & Zahn, 1992) which must influence the thermal balance of the layer. In particular, the level at which the layer becomes unstable is likely to be perturbed by the flow, causing the shape of the base of the convection zone to deviate from a sphere. Of course, centrifugal force imparts an oblateness to level surfaces, but that is very small: it induces a quadrupole component to the angular distribution of pressure of order $r_b^3 \Omega_b^2/GM \approx 8 \times 10^{-5}$, where r_b and Ω_b are respectively the radius and characteristic angular velocity of the tachocline. This contributes to the even component of the frequency splitting of p-mode multiplets by a similar amount (when the splitting is expressed in units of the multiplet frequency). For a 3 mHz mode, for example, the centrifugal contribution to the difference between the zonal frequency and the corresponding mean sectoral frequency is approximately 20 nHz (e.g. Gough & Thompson, 1990). However, the position of the lower boundary of the convection zone depends on the relatively small difference between two gradients, and the level at which it vanishes could suffer much more severe distortion.

One should bear in mind also that in the tachocline there may be a magnetic field intense enough to modify the convection significantly, by either inhibiting or enhancing the flow. There are models of the solar cycle that rely on the intensification by the shear of a belt, or belts, of kilogauss toroidal field. Such belts might partially suppress the convection, imparting a corrugation in its lower boundary. As the field strengthens, it becomes buoyant, and from time to time parts of it break away, and rise through the convection zone, subsequently to reach the surface of the sun and partake in the magnetic activity visible in the atmosphere. In this phase, convection is presumably enhanced by the field. It is not unlikely, therefore, that not only is the interface between the convection zone and the relatively quiescent interior aspherical, but that its shape changes with time during the cycle.

It is manifestly of interest to seek direct evidence for such asphericity. The implications concern not solely the magnehydrodynamics of the tachocline itself, but the dynamics of the entire sun, and it hydrostatic and thermal stratification. It could provide a clue to the origin of the solar-cycle variation in the solar constant, whether it be due principally to a variation in the solar luminosity or to a latitudinal redistribution of the radiative flux. It has not yet been established whether the cause of the variation is situated in the upper boundary layer of the convection zone or whether it is more deeply seated. If it is the latter, the base of the convection zone would need to vary by perhaps 0.1 pressure scale heights, which might be detectable seismologically.

2. INVERSION METHOD AND RESULTS

The temperature gradient is almost adiabatic through the convection zone, but it changes sharply at the base of the zone becoming subadiabatic in the radiative interior. The location of this transition coincides with the

sharp variation of the slope of the relative difference in the quantity $u = p/\rho$ between the Sun and a solar model, provided that the zone of adiabatic convection is deeper in the model than in the Sun (see Figure 1).
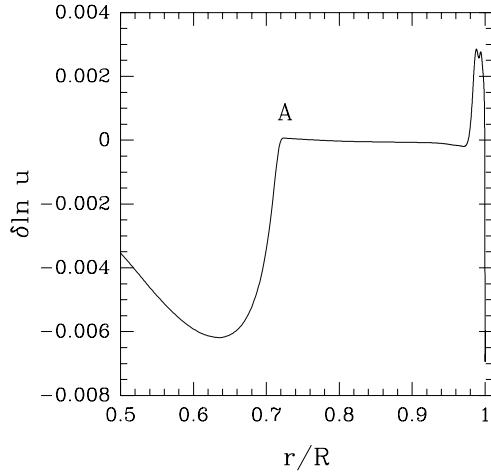
Figure 1: *Relative difference in $u = p/\rho$ between two solar models. Point A shows the location of the base of the shallower convection zone.*

In the case of axisymmetric perturbations to the solar structure, the frequency variations are given by (e.g. Gough, 1993)

$$\frac{\delta\omega_{nlm}}{\omega_{nl}} = \sum_{\lambda} Q_{\lambda lm} \int_0^R \left(K_{u,Y}\delta\ln u_\lambda + K_{Y,u}\delta Y_\lambda \right) dr, \quad (1)$$

where

$$Q_{\lambda lm} = \int_{-1}^1 [P_l^m(\mu)]^2 P_\lambda(\mu) d\mu; \quad (2)$$

P_l^m are normalized Legendre functions; P_λ are Legendre polynomials; the angular dependence of the perturbation $\delta\ln u$ is considered to have been expanded in the form:

$$\delta\ln u(r,\mu) = \sum_{\lambda>0} \delta\ln u_\lambda(r)P_\lambda(\mu), \quad (3)$$

and similarly, the perturbation to the helium abundance Y; $K_{u,Y}(r)$ and $K_{Y,u}(r)$ are the kernel functions of the perturbations, which have been obtained from a variational principle.

We have carried out the two-dimensional inversion of the data obtained by Woodard & Libbrecht (1993) in the form:

$$\delta\omega_{nlm} = (l+1/2)\sum_{k=1}^{12} a_k(n,l)P_k\left(\frac{m}{l+1/2}\right) \quad (4)$$

using an asymptotic expansion of the integral $Q_{\lambda lm}$ (e.g. Kosovichev, 1988) and a modified Backus-Gilbert inversion technique.

Figure 2 shows the localized averages of δu at three colatitudes: $\theta = 0$, $\pi/4$ and $\pi/2$.
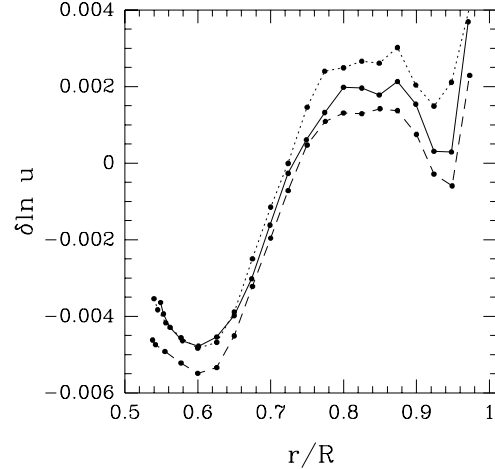
Figure 2: *Relative difference in $u = p/\rho$ between the sun and a standard model at colatitude $\theta = 0^o$ (solid curve), 45° (dashed), and 90° (dotted)*

The results reveal systematic differences in radial location of the curves at about 0.7 R, which can be interpreted as variations of the depth of the convection zone. The absolute measurement of the depth by this technique is not very precise because the widths of the averaging kernels localized in this area are larger than the radial displacement. However, the displacement of the curves gives a measure of the relative variations of the depth. They do not exceed 0.02 R and, therefore, are consistent with the theoretical expectations.

3. CONCLUSION

We have determined the difference $\delta u/u$ as a function of radius at various latitudes between the Sun and a spherically symmetrical solar model by inverting the BBSO data (Woodard & Libbrecht, 1993). The results offer evidence that the convection zone may be somewhat deeper at the equator than it is at the poles. The variation of the depth, however, does not exceed $0.02R_\odot$.

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