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Helioseismic Measurements of Elemental Abundances in the Sun's Interior

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Abstract. The method of 'secondary' helioseismic inversion, in which the equations of state and the thermal balance are used in addition to the hydrostatic equations, have been applied to determine the helium abundance in the Sun's interior. By inverting the observed p-mode frequencies, direct evidence for gravitational settling of helium has been obtained. The helium abundance in the convection zone inferred from the BBSO data (Libbrechh *et al.*, 1990) using the OPAL equation of state (Rogers & Iglesias, 1993) is 0.248 ± 0.006.

Key words: solar oscillations, helioseismic inversion, helium abundance

1. Introduction

Helioseismology provides a unique tool for investigating chemical composition of the interior of the Sun. There are two basic ways of obtaining the information. The first calibrates theoretical solar models by comparing either the observed oscillation frequencies with the eigenfrequencies of the models, or the primary seismic parameters (e.g. the sound speed, the density and the adiabatic exponent) inverted from the observed frequencies with the corresponding parameters of the solar models.

The second approach measures abundances by direct ('secondary') inversions of the frequencies, incorporating additional equations of the stellar structure into the helioseismic inverse problem. The additional equation to estimate composition of the convection zone is the equation of state that relates variations of the adiabatic exponent in zones of ionization of elements to their abundances. In the radiative interior where the most abundant elements are almost totally ionized, the energy equations together with equations of the energy generation rate and the opacity are used to relate primary seismic parameters with abundances.

2. The helioseismic inversion method

For the frequency inversions, the linearized integral equations relating frequency perturbations, $\delta\nu_i$, to variations of the solar structure are derived from a variational principle. They are transformed to depend on a chosen pair of deviation variables $\boldsymbol{f}$ (e.g. $\boldsymbol{f}= (\delta \ln \rho, \delta \ln \gamma)$ that are assumed to be functions of radius, r, alone (e.g. Gough & Kosovichev, 1988), yielding

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{\mathrm{R}_\odot} \boldsymbol{K}_{\boldsymbol{f}} \cdot \boldsymbol{f}\, \mathrm{d}r + \frac{F(\nu_i)}{E_l(\nu_i)}, \quad i = 1, ..., N. \tag{1}$$

Here $\delta\nu_i$ is the frequency difference between the eigenfrequency ν_i of a solar model and the corresponding frequency of the Sun, $\mathrm{R}_\odot$ is the radius of the Sun, and $E_l(\nu_i)$ is the mode inertia. The arbitrary function $F(\nu_i)$ is added to take into account surface effects. The subscript i labels the modes; N is the total number of modes in a data set.

The structure parameters $\boldsymbol{f}$ can be of two types: 'primary', e.g. the density, ρ, and the adiabatic exponent ,γ, or 'secondary', e.g. the hydrogen abundance, X, and the heavy element abundance, Z. For the 'primary' parameters Eq. (1) is derived under the basic assumptions about the solar structure: spherical symmetry and hydrostatic equilibrium; whereas for the 'secondary' parameters additional structure equations must be considered. Only two structure variables appear in Eq. (1) because in deriving them the equations of hydrostatic support

$$\frac{dp}{dr} = -\frac{Gm\rho}{r^2} \tag{2}$$

$$\frac{dm}{dr} = 4\pi\rho r^2 \tag{3}$$

have been imposed.

Two main options for the 'secondary' parameters have been studied. The first includes the equation of state in the form $\gamma = \gamma(p, \rho, X_j)$, where X_j are element abundances. Since the variations of the adiabatic exponent, γ, occur mainly in the zones of ionization of helium and hydrogen at the top of the convection zone, the frequency variations can be expressed in terms of the uniform helium abundance in the convection zone (assuming that the abundances of the heavier elements are known) and one of the hydrostatic variable, e.g. density or the ratio $u = p/\rho$.

The second option is to consider a full set of structure equations, including the equations of thermal balance and of energy transport, thus introducing 'non-seismic' variables, such as temperature, elemental abundance and opacities in the radiative zone, into the seismic equations.

Equations (1) can be transformed into a relation between $\frac{\delta\nu_i}{\nu_i}$ and composition deviations δX and δZ by additionally imposing the constraint of thermal balance through the equations

$$\frac{dL}{dr} = 4\pi\rho r^2\epsilon, \tag{4}$$

$$\frac{dT}{dr} = \begin{cases} \frac{-3\kappa\rho L}{64\pi\sigma r^2 T^3} & \text{in radiative zones} \\ \left(\frac{dT}{dr}\right)_c & \text{in the convection zone} \end{cases} \quad (5)$$

where L is luminosity, T is temperature, ϵ is the energy-generation rate per unit mas, κ is opacity and σ is the Stefan-Boltzmann constant. The function $(dT/dr)_c$ is provided by the mixing-length formalism relating temperature gradient to energy transport in the convection zone. In addition, the functions of $\epsilon(\rho, T, X, Z)$ and $\kappa(\rho, T, X, Z)$ and their partial derivatives are required. The transformation then yelds new perturbation relations which may be written

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{\mathrm{R}_\odot} \boldsymbol{K_g} \cdot \boldsymbol{g}\, \mathrm{d}r + \frac{F(\nu_i)}{E_l(\nu_i)}, \quad i = 1, ..., N \quad (6)$$

where $\boldsymbol{g} = (\delta X, \delta Z)$.

It is a straightforward matter to compute the kernels in equation (6). The linearized structure equations (2-5) formally can be written

$$\mathcal{A}\boldsymbol{f} = \boldsymbol{g}, \quad (7)$$

where $\mathcal{A}$ is a linear differential operator. The equation must be supplemented with appropriate boundary conditions, that are derived from the requirement that conditions in the photosphere are unchanged, which we adopt in the form $\ln\rho = 0$, $\ln L = 0$ at $r = R$. Substituting equation (7) into equation (6) yields

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{\mathrm{R}_\odot} \boldsymbol{K_g} \cdot \mathcal{A}\boldsymbol{f}\, \mathrm{d}r + \frac{F(\nu_i)}{E_l(\nu_i)} = \int_0^{\mathrm{R}_\odot} \mathcal{A}^* \boldsymbol{K_g} \cdot \boldsymbol{f}\, \mathrm{d}r + \frac{F(\nu_i)}{E_l(\nu_i)}, \quad (8)$$

where $\mathcal{A}^*$ is the adjoint of $\mathcal{A}$ By demanding that the expressions on the right-sides of equations (1) and (8) be identical, we obtain

$$\mathcal{A}^* \boldsymbol{K_g} = \boldsymbol{K_f}. \quad (9)$$

Thus, the seismic kernels for the element abundances are obtained as solutions of the adjoint linearized structure equations.

The inversion procedure basically consists of constructing linear combinations of Eqs (1) or (6) for a set of observed modes, which provide localized averages of the structure parameters $\boldsymbol{f}$ and $\boldsymbol{g}$:

$$\overline{\boldsymbol{f}}(r_0) = \int_0^{\mathrm{R}_\odot} A_{\boldsymbol{f}}(r_0, r) \boldsymbol{f}\, \mathrm{d}r, \qquad \overline{\boldsymbol{g}}(r_0) = \int_0^{\mathrm{R}_\odot} A_{\boldsymbol{g}}(r_0, r) \boldsymbol{g}\, \mathrm{d}r. \quad (10)$$

3. The inversion results

The inverted data used in this analysis are combinations of 16 frequencies of low-degree modes (l = 0, 1 and 2) in the frequency range $2.5 \lesssim \nu \lesssim 3$ mHz taken from the IPHIR data set (Toutain & Fröhlich, 1992) and 598 frequencies of intermediate-degree modes (l = 4–140, ν = 1.5–3 mHz) observed at BBSO in 1988 (Libbrecht *et al.*, 1990).

The estimates of the helium abundance in the convection zone depend entirely on the variation of γ in the HeII ionization zone and are obtained using the equation of state. A version of the 'MHD' equation of state (Däppen *et al.*, 1988) gives the helium abundance $Y \approx 0.232 \pm 0.006$, whereas the OPAL equation of state (Rogers & Iglesias, 1993) leads to $Y \approx 0.254 \pm 0.006$ (Fig. 1a). The most recent version of the OPAL equation of state gives a slightly lower value: 0.248 ± 0.006. The difference between the MHD and OPAL equations of state has not been fully resolved. However, both estimates are substantially lower than the standard model value, $Y = 0.280$, and therefore, consistent with the general picture of the helium settling.

Further evidence for gravitational settling of helium is given by direct inversion for the helium abundance Y in the radiative zone, showing that, compared with the standard model, helium is underabundant outside the solar core (Fig. 1b). Details of the interior

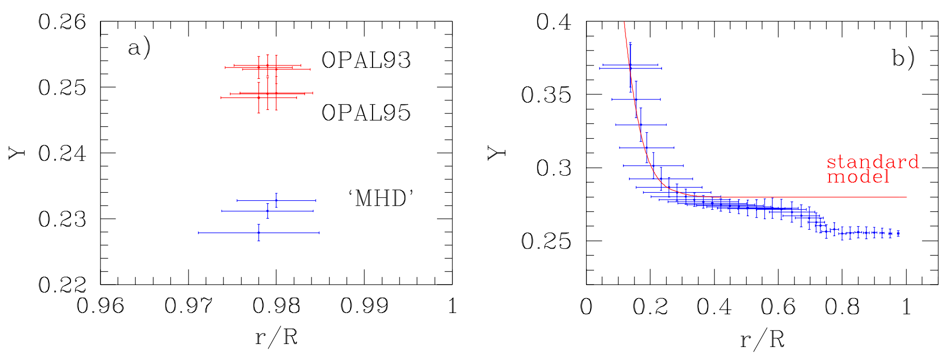

Figure 1: Optimally localized averages of the helium abundance Y in the Sun, inferred from the combination of BBSO and IPHIR data. The solid curve shows the abundance of helium in the standard solar model

structure such as material mixing in the core, convective overshoot, diffusion of heavy elements and the role of turbulence have yet to be established. Their determination requires more accurate measurements of the frequencies of solar oscillations.

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