

## Determination of 3D Internal Structure and Flows by Tomographic Inversion

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**Abstract.** I present initial results of inversion of travel-time maps recently obtained by Duvall *et al.* (1995). The maps represent measurements of the time for acoustic waves to travel between points on a solar surface and surrounding annuli. The measurements are sensitive to perturbations of the sound speed and flows along the ray paths. A 3D inversion method based on Fermat's Principle and a conjugate-gradient technique have been applied to infer the sound speed and the velocity of flows from the observations obtained at the South Pole 4-5 Jan. 1991. The spatial resolution of the inversion is 1.75 degree in both longitude and latitude, and 15 Mm in depth. The results reveal large-scale subsurface structures and flows related to the active regions that are important for understanding the physics of solar activity and large-scale convection.

*Key words:* helioseismology, convection zone, travel time, inversion

### 1. Introduction

Duvall *et al.* (1995, 1996) have provided maps of travel times of sound waves propagating in local regions in the convection zone. The measurements were obtained by estimating cross-correlation functions of fluctuations of brightness of the solar surface caused by 5-minute acoustic oscillations. The waves are stochastically excited by granulation uniformly distributed on the surface; therefore, there are waves propagating between any two points on the surface. However, because of the high level of noise, the data are averaged over large surface areas. Typically, the measurements represent a mean travel time between a central point and surrounding annuli, which were chosen in four radial distance ranges,  $\Delta$ : 2.5–4.25, 4.4–7, 7.25–10, and 10.25–15 heliographic degrees ( $1^\circ \approx 12.15$  Mm). The acoustic rays that propagate to these radial distances span the upper 30% of the convection zone, penetrating to a depth,  $z$ , of about 64 Mm (Fig. 1a). Duvall *et al.* have obtained two sets of data. The first represents the mean of the travel times

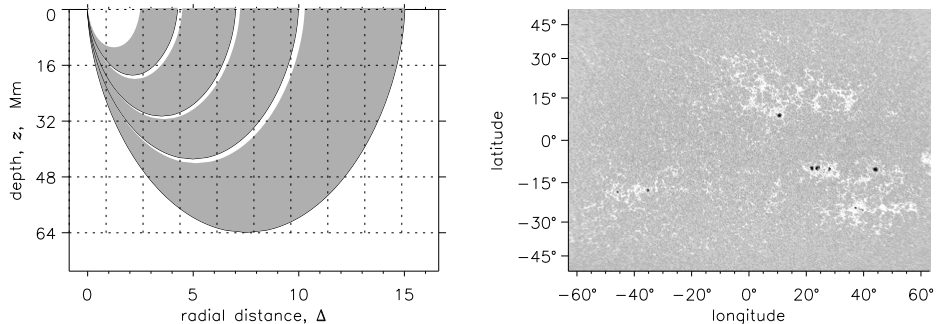


Figure 1: a) The regions of ray propagation (shaded areas) corresponding to the measurements by Duvall *et al.* (1995), as a function of depth,  $z$ , and the radial distance,  $\Delta$ , from a point on the surface. The rays are also averaged over a circular regions on the surface, forming three-dimensional figures of revolution. The dashed line are the boundaries of the cells of the inversion model; b)  $\text{Ca}^+\text{K}$ -line image of the observed area.

for the rays propagating from a central point to the surrounding annuli ( $\tau^+$ ) and for the rays propagating in the opposite direction ( $\tau^-$ ). The second set is the difference of these travel times. The mean travel time,  $0.5(\tau^+ + \tau^-)$ , depends primarily on the sound speed along the ray paths, while the difference,  $\tau^+ - \tau^-$ , is sensitive to the flow velocity.

We present an inversion technique to infer the perturbations of the sound speed and flow velocities from the data. The technique is similar to seismic tomography (e.g. Izer & Hirahara, 1993); however, it is more complicated computationally because each data point is no longer a 1D line integral but an average over  $\sim 10^3$  ray paths that sample a large 3D region (Fig. 1a).

## 2. The inversion method

The travel time,  $\tau_i$ , along a single ray path,  $\Gamma_i$ , in the geometrical optics approximation can be expressed in terms of the sound speed (wave group velocity),  $c(\mathbf{x}, t)$ , and the flow velocity  $\mathbf{v}(\mathbf{x}, t)$ :

$$\tau_i(t) = \int_{\Gamma_i} \frac{ds}{c(\mathbf{x}, t) + \mathbf{v}(\mathbf{x}, t) \cdot \mathbf{n}}, \quad (1)$$

where  $\mathbf{x}$  is the spatial coordinate,  $t$  is the time,  $s$  is the distance along the ray and  $\mathbf{n}$  is a unit vector tangent to the ray. The sign of  $\mathbf{v} \cdot \mathbf{n}$  depends on the direction of propagation; therefore, the travel time in opposite directions differs due to the effects of flows. Since variations of the travel time in the area of observation (Fig. 1b) do not exceed 5%, the effects of flow and sound-speed inhomogeneity can be considered, in the first approximation, as perturbations to a horizontally uniform static reference model with a sound speed  $c_0(\mathbf{x})$ . Then, the reference travel time is  $\tau_{0,i} = \int_{\Gamma_{0,i}} ds/c_0(\mathbf{x})$ . Fermat's Principle applied to the variation of the travel time gives

$$\delta\tau_i^\pm(t) \equiv \tau_i^\pm(t) - \tau_{0,i} = - \int_{\Gamma_{0,i}} \frac{\delta c(\mathbf{x}, t) \pm \mathbf{v}(\mathbf{x}, t) \cdot \mathbf{n}}{c_0^2(\mathbf{x})} ds, \quad (2)$$

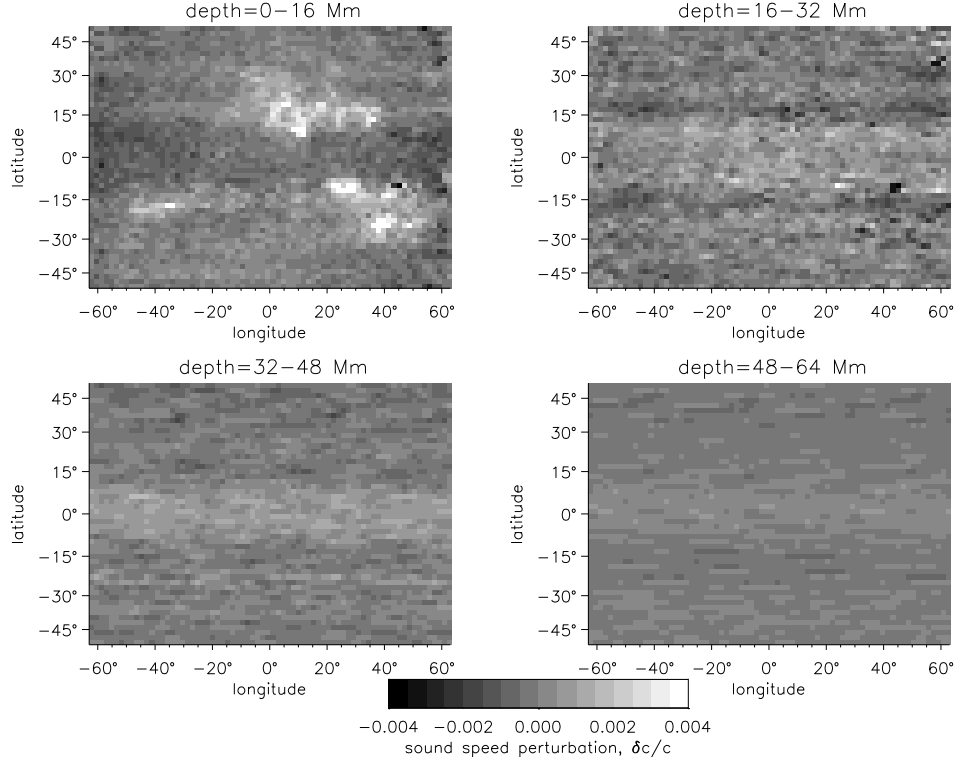


Figure 2: Variations of the sound speed in the four-layer model of the upper convection zone relative to a horizontally uniform reference solar model.

where ‘+’ is for the rays propagating from the central point,  $\Delta = 0$ , to the annuli, and ‘-’ - for the rays in the opposite direction. The mean and the difference between the travel time residuals are

$$\delta\tau_i^{\text{mean}}(t) = - \int_{\Gamma_{0,i}} \frac{\delta c(\mathbf{x}, t)}{c_0^2(\mathbf{x})} ds, \quad \delta\tau_i^{\text{diff}}(t) = -2 \int_{\Gamma_{0,i}} \frac{\mathbf{v}(\mathbf{x}, t) \mathbf{n}}{c_0^2(\mathbf{x})} ds. \quad (3)$$

We assume that  $\delta c$  and  $\mathbf{v}$  did not change during the 8 hours of observation and represent them by discrete models dividing the 3D region of wave propagation that covers the area  $123^\circ \times 106^\circ$  and is 64 Mm deep into  $72 \times 52 \times 4$  rectangular cells. We also assume the relative perturbations of the sound speed and the ratio of the flow velocity to the sound speed are constant within the cells:

$$\frac{\delta c}{c}(\mathbf{x}, t) = \sum_{ijk} \left( \frac{\delta c}{c} \right)_{ijk}, \quad \frac{\mathbf{v}}{c}(\mathbf{x}, t) = \sum_{ijk} \left( \frac{\mathbf{v}}{c} \right)_{ijk} \quad (4)$$

Computing the integrals (3) and averaging them over the 3D regions of ray propagation for each of the four radial distance intervals, we obtain two systems of linear equations which relate the data to the sound speed variation and to the flow velocity. The equations have been solved by a regularized least-squares technique.

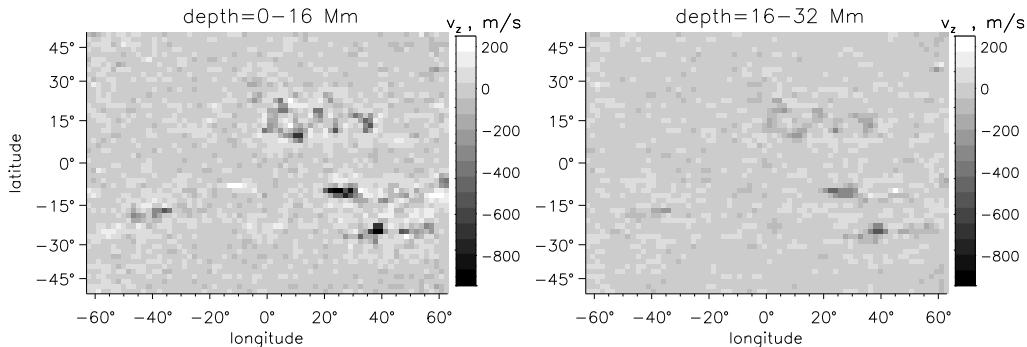


Figure 3: Vertical flow velocity in the convection zone.

### 3. The inversion results

The results of inversion are shown in Figures 2 and 3 as grey-scale maps of the sound-speed perturbation,  $\delta c/c$ , and the vertical velocity,  $v_z$ , in the four layers of our discrete model. They can be compared with the Ca<sup>+</sup>K-line intensity map in Fig. 1b. The most interesting detail of the sound-speed map is the equatorial belt of lower sound speed ( $\delta c/c \sim -5 \times 10^{-4}$ ) in the upper layer and higher speed in the lower layers. The belt is most pronounced in the second layer ( $\delta c/c \sim +6 \times 10^{-4}$ ), where it is surrounded by narrow regions of lower sound speed at latitudes  $\approx \pm 15^\circ$ . This could be evidence of the thermal shadow due to a band of magnetic field in the convection zone (Parker, 1987; Kuhn, *et al.*, 1988).

The perturbations of the active regions can be seen in the two upper layers, but not deeper. The vertical velocity maps show strong downflows of  $\approx 1$  km/s around sunspots, and some much weaker isolated upflows at the boundaries of the equatorial belt that may be related to emerging magnetic structures.

The spatial resolution of the data is insufficient to study the structure and dynamics of individual sunspots. However, it is interesting to note that the peak of the downdraft velocity occurs between the spots in the bipolar group located at latitude  $-10^\circ$  and longitude  $25^\circ$ . This group results in two separate strong positive sound-speed perturbations in the top layer (0-16 Mm), which merge into a single perturbation in the second layer (16-32 Mm). A detailed analysis of the inversion results is published elsewhere.

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