A MECHANISM OF HELICITY VARIATIONS ON THE SUN

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Abstract. Helicity of solar magnetic fields plays an important role in dynamo theories of the solar cycle. The helicity has been known to vary with the main 11-year period (Hale's cycle). Recent observations have revealed significant helicity variations on a shorter time scale, with a characteristic period of approximately 2 years. We suggest an explanation for the observed variations of the magnetic helicity, based on our model of the double magnetic cycle of solar activity. The quasi-biennial variations of the helicity are the consequence of the influence of erupted magnetic fields of the main cycle on the helicity in the regions of generation of the high-frequency component of magnetic field. This model suggests that the low-frequency component is generated at the base of the convective zone due to large-scale radial shear $\partial\Omega/\partial r$ of angular velocity Ω. The high-frequency component may be generated in the subsurface region due to latitudinal shear $\partial\Omega/\partial\theta$ or due to the radial shear in this region.

1. Introduction

Kinetic helicity which is defined as the product of velocity **v** of a flow and its own vorticity $\nabla \times \mathbf{v}$ can be a source of magnetic field in a plasma with gyrotropic turbulence, where an ensemble average of the helicity $\langle \mathbf{v} \cdot (\nabla \times \mathbf{v})\rangle$ is nonzero (Krause and Steenbeck, 1967). The mechanism of the generation of magnetic field is very simple. The Coriolis force causes convection to be cyclonic. That means that a rising element of fluid rotates and transforms magnetic force lines of the azimuthal component of the field into loops with a non-vanishing meridional component. A large number of such loops may coalesce and generate a large-scale dipole field (Parker, 1993). Krause and Steenbeck (1967) called the turbulent electromotive force, ε, creating such a poloidal magnetic field the 'α-effect'. The 'α-effect' is able to generate toroidal current

$$\mathbf{j} = \sigma\varepsilon = \sigma\alpha\mathbf{B_T}, \quad (1)$$

where α is proportional to the kinetic helicity: $\alpha = \alpha_h = -\langle \mathbf{v}\cdot(\nabla\times\mathbf{v})\rangle\tau/3$, τ is the turbulence correlation time, σ is the electrical conductivity, and $\mathbf{B_T}$ is the toroidal magnetic field. Therefore, this effect generates poloidal magnetic field $\mathbf{B_P}$ from the toroidal field ($\mathbf{B_T} \rightarrow \mathbf{B_P}$). The dynamo mechanism is completed by adopting that the toroidal magnetic field is created by differential rotation or by the 'α-effect' itself.

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Further investigations have shown that magnetic field can affect the helicity causing its variations and generating new 'magnetic helicity.' The magnetic helicity is usually defined as $\int_{V_m} \mathbf{A}\cdot\mathbf{B}\, \mathrm{d}^3x$, where V_m is the volume of a region such as $\mathbf{nB} = 0$ on its surface. $\mathbf{B}$ is the magnetic field strength, and $\mathbf{A}$ is the vector potential. The value $h_m = \mathbf{A} \cdot \mathbf{B}$ is the density of magnetic helicity. We will refer to this quantity as the magnetic helicity. According to Kliorin and Ruzmaikin (1982) coefficient α in Equation (1) can be represented as a sum of two components: kinetic α_h and magnetic $\alpha_m = \langle \mathbf{b} \cdot (\nabla \times \mathbf{b})\rangle / 3\mu_0\rho$, which is caused by the fluctuating component of the magnetic field, $\mathbf{b} = \mathbf{B} - \langle \mathbf{B} \rangle$. Here $\langle \mathbf{B} \rangle$ is the ensemble averaged (mean) magnetic field, ρ is the plasma density, and μ_0 is the magnetic constant.

Coefficient α_m is proportional to an ensemble average of the fluctuations of magnetic helicity, $\langle \mathbf{a} \cdot \mathbf{b} \rangle$, where $\mathbf{a} = \mathbf{A} - \langle \mathbf{A} \rangle$, and $\mathbf{b} = \mathbf{B} - \langle \mathbf{B} \rangle$ (Vainshtein, 1983). If the total helicity is conserved then $\langle \mathbf{A}\cdot\mathbf{B} \rangle = \langle \mathbf{A} \rangle \cdot \langle \mathbf{B} \rangle + \langle \mathbf{a}\cdot\mathbf{b} \rangle$ = const. That means that time variations of α_m are proportional to the variations of $\langle \mathbf{A} \rangle \cdot \langle \mathbf{B} \rangle$. The last quantity, in principle, can be obtained from measurements of the mean magnetic field, thus providing an important insight in the dynamo mechanism.

Numerous investigations have been devoted to studying the magnetic helicity on the Sun (e.g., Seehafer, 1990; Martin, 1992; Rust and Kumar, 1994; Pevtsov, Canfield, and Metcalf, 1995; Abramenko, Wang, and Yurchishin, 1996; Zhang and Bao, 1998). Unfortunately, direct observations of magnetic helicity $\mathbf{A} \cdot \mathbf{B}$ are not possible because vector potential $\mathbf{A}$ is not measured. However, for practical purposes magnetic helicity can be characterized by a force-free field approximation (Pevtsov, Canfield, and Metcalf, 1995) or by measured current helicity $h_c = \langle B_z \rangle (\nabla \times \langle \mathbf{B} \rangle)_z$ (Abramenko, Wang, and Yurchishin, 1996), which varies in a similar way as the magnetic helicity (Seehafer, 1990).

Recently, Bao and Zhang (1998) have found that the magnetic (current) helicity observed during the solar cycle 22 (1988–1997) varied with time. The first maximum in the current helicity was in 1991, the second was in 1989 June, and the third in 1990 (see Figure 3 in Bao and Zhang, 1998). These variations could be interpreted as a quasi-biennial periodicity. Therefore, the observations indicate two main periodicities of helicity variations on the Sun: 11 yrs and 1.5–2.5 yrs.

The existence of the 11-yr periodicity is easy to understand (Ruzmaikin, 1996). In fact, both toroidal and poloidal magnetic fields change with a period approximately equal to 22 yr. Therefore, if $\langle B \rangle \sim \sin \omega t$ and $\langle A \rangle \sim \cos \omega t$, then $\langle A \rangle \langle B \rangle \sim \sin 2\omega t$, that is the period of the corresponding helicity variations is 11 yr. However, the 2-yr variations of the helicity are a puzzle.

In this paper we try to clarify this situation and suggest an explanation for the quasi-biennial periodicity in magnetic helicity.

2. A Model of Helicity Variations

Our explanation of the observed helicity variations is based on the previously proposed phenomenological model of the double magnetic cycle on the Sun (Benevolenskaya, 1998). We argued that solar magnetic cycles are complicated and, in fact, consist of two quasi-periodic cycles: a low-frequency component (22-yr Hale's cycle) and a high-frequency (quasi-biennial) component.

The low-frequency component can be generated at the base of the convective zone due to large-scale radial shear $\partial\Omega/\partial r$ of angular velocity Ω. The high-frequency component may be generated in subsurface regions due to latitudinal shear $\partial\Omega/\partial\theta$ or due to a radial shear. The recent investigations of solar interior rotation by helioseismology have shown significant radial gradients of the angular velocity at the top and bottom of the convective zone together with the latitudinal gradient of the angular velocity in the top layer (Schou *et al.*, 1998). For simplicity we use only the latitudinal shear for generation of the high-frequency component. We employ Cartesian coordinates x, y and z denoting the radial, azimuthal and latitudinal coordinates respectively, and consider axisymmetrical solutions. We use subscripts 1 and 2 to denote the properties of the low-frequency and high-frequency components respectively, and omit the angular brackets in the notations of the mean-field variables.

At the base of the convection zone turbulence is suppressed by a strong magnetic field (Parker, 1993) and, therefore, diffusivity in the first layer, η_1, could be less then diffusivity η_2 in the second layer. The axisymmetrical mean magnetic field is decomposed into toroidal and poloidal parts, and represented by the azimuthal component of the vector-potential, A, and the toroidal component of the field strength, B. These characteristics of the magnetic field are described by two systems of non-linear differential equations, each of them describing the evolution of the two independent sources of the magnetic field. The equation for α_m was obtained by Kliorin and Ruzmaikin (1982), and, generally, can be written as $\partial\alpha_m/\partial t = -\nu\alpha_m + pAB$, where $\nu > 0$ and $p < 0$ are parameters which depends on properties of the plasma turbulence. Then, the two system are:

$$\begin{cases} \dfrac{\partial A_1}{\partial t} = (\alpha_{h1} + \alpha_{m1})B_1 + \sigma\Delta A_1 \,, \\ \dfrac{\partial B_1}{\partial t} = -\dfrac{\partial A_1}{\partial z} G_x + \sigma\Delta B_1 \,, \\ \dfrac{\partial \alpha_{m1}}{\partial t} = -\nu\alpha_{m1} + pA_1B_1 \,, \end{cases} \tag{2}$$

$$\begin{cases} \dfrac{\partial A_2}{\partial t} = (\alpha_{h2} + \alpha_{m2})B_2 + \Delta A_2 \ , \\ \dfrac{\partial B_2}{\partial t} = \dfrac{\partial A_2}{\partial x} G_z + \Delta B_2 \ , \\ \dfrac{\partial \alpha_{m2}}{\partial t} = -\nu\alpha_{m2} + pA_2B_2 \ , \end{cases} \tag{3}$$

where α_{h1} and α_{h2} are the coefficients proportional to the kinetic helicity in the two regions, and $\sigma = \eta_1/\eta_2$ is the ratio of diffusivity in these regions.

If the two sources of magnetic field were independent then in the frame of this model it is difficult to explain the observed variations of the high-frequency components of the magnetic field and helicity. Therefore, as in our previous paper (Benevolenskaya, 1998) we assume that the erupted low-frequency magnetic field can influence the physical conditions in the region where the high-frequency component operates, through modifying helicity there. This allows the high-frequency component to vary with time. In this case, the equation for the variable part of coefficient α_{m2} (which is proportional to the magnetic helicity) in the region of generation of the high-frequency component becomes

$$\frac{\partial \alpha_{m2}}{\partial t} = -\nu\alpha_{m2} + p(A_2 + aA_1)(B_2 + aB_1) \ , \tag{4}$$

where parameter a represents the influence of the low-frequency component of magnetic field, B_1, on the helicity in region 2 where the high-frequency component is generated.

Since there is an underlying periodicity in all calculations, we use this period as a relative unit of time, and employ the following transformations:

$$\begin{aligned} & t \longrightarrow t' = \eta_2 k^2 t \alpha_0 \ , && \alpha_h \longrightarrow \alpha_h' = \alpha_h/\eta_2 k^2 \alpha_0 \ , \\ & \sigma = \eta_1/\eta_2 \ , && \alpha_m \longrightarrow \alpha_m' = \alpha_m/\eta_2 k^2 \alpha_0 \ , \\ & G_z \longrightarrow G_z' = G_z/\eta_2 k \alpha_0 \ , && G_x \longrightarrow G_x' = G_x/\eta_2 k \alpha_0 \ , \end{aligned}$$

where α_0 is a characteristic mean value of α, and k is the radial wave number. With these transformations, G_x and G_z represent the dynamo numbers in our model. The period of the main cycle, 22 years, is chosen as the unit of time.

Following Weiss, Cattaneo, and Jones (1984) the solutions of these equations were found as $A_1 = a(t)e^{ikz}$, $B_1 = b(t)e^{ikz}$ and $\alpha_m = C_0(t) + C(t)e^{2ikz}$, Im $C_0 = 0$ for x_1 level, and the same expressions for x_2 level where z is replaced by x. A, B, C are complex functions as $A = a_1 + ia_2$, $B = b_1 + ib_2$.

Therefore, set of the partial differential equations (2)–(3) is reduced to the following equations, which were then solved numerically:

$$\begin{cases} \dot{A}_1 = (\alpha_1 + C_{10})B_1 - \sigma A_1 + \frac{1}{2} B_1^* \ C_1 \ , \\ \dot{B}_1 = -iA_1^* \ G_x - \sigma B_1 \ , \\ \dot{C}_1 = -\nu C_1 + pA_1B_1 \ , \end{cases} \tag{5}$$

$$\begin{cases} \dot{A}_2 = (\alpha_2 + C_{20})B_2 - A_2 + \frac{1}{2}B_2^* C_2 , \\ \dot{B}_2 = iA_2G_z - B_2 , \\ \dot{C}_2 = -\nu C_2 + p(A_2 + aA_1)(B_2 + aB_1) . \end{cases} \quad (6)$$

If $a = 0$ then these two systems are independent, and for the low-frequency component described by system (5) we obtain 11-yr variations for coefficient α_{m1} as described in Introduction. For the high-frequency component in this case we find coefficient $\alpha_{m2} \sim \sin 2\omega_2 t$ that gives 1-yr variations if the magnetic field varies with a 2-yr period (frequency ω_2), because $B_2 \sim \sin \omega_2 t$ and $A_2 \sim \cos \omega_2 t$ and $\omega_2 = 2\pi/2$ yr. However, if these systems are coupled ($a \neq 0$) due to the influence of the low-frequency magnetic field on helicity in the high-frequency zone then the 2-yr periodicity in helicity can be obtained because of the non-linear cross-terms, aA_1B_2 and aA_2B_1.

3. Results

We have investigated solutions of the two non-linear systems for the coupling parameter, a, in the interval $0 \leq a \leq 2$, and $\alpha_{h1} = \alpha_{h2} = 1$, $\nu = 0.5$, $p = -1$, $\sigma = 0.1$, $G_x = 0.3$, $G_z = 20$, $C_{10} = C_{20} = 0$, which are in the reasonable range for the Sun (Benevolenskaya, 1998).

In the case $a = 0$ (no coupling) coefficient α_{m2} shows the dominant variations with the frequency ≈ 20 (Figures 1(a) and 1(b)). Since our frequency unit is 1/22 yr^{-1}, this frequency corresponds to a 1-year period. Multiple low-frequency peaks in Figure 1(b) at low frequencies represent some non-linear noise. The total vector-potential, A (Figures 1(c) and 1(d)) has two main frequencies, ≈ 1 and ≈ 11, corresponding to periods 22 yr and 2 yr. Therefore, the solution with no coupling between the two dynamo zones shows the quasi-periodic variations of magnetic field, corresponding to the observations (Benevolenskaya, 1998), but gives the incorrect period of the helicity variations in the upper layer (1 year instead of the observed 2 years).

With the coupling, when the low-frequency component of the magnetic field generated at the bottom of the convection zone influences the helicity in the upper layer, the helicity variations become more regular and clearly show two components with the frequencies ≈ 2 and ≈ 10, which correspond to 11-yr and 2-yr periods in accordance with the observations (Figures 2(a) and 2(b)). The magnetic field variations in this case also have correct periods of ≈ 22 and ≈ 2 years, with the second component being much weaker. However, the amplitudes of these components in the helicity variations are comparable, in agreement with the observations of Bao and Zhang (1998).

When the influence of the low-frequency mode becomes stronger the high-frequency component in the helicity variations decreases. This situation is illustrated in Figure 3 which shows the results for $a = 0.2$. This may explain why

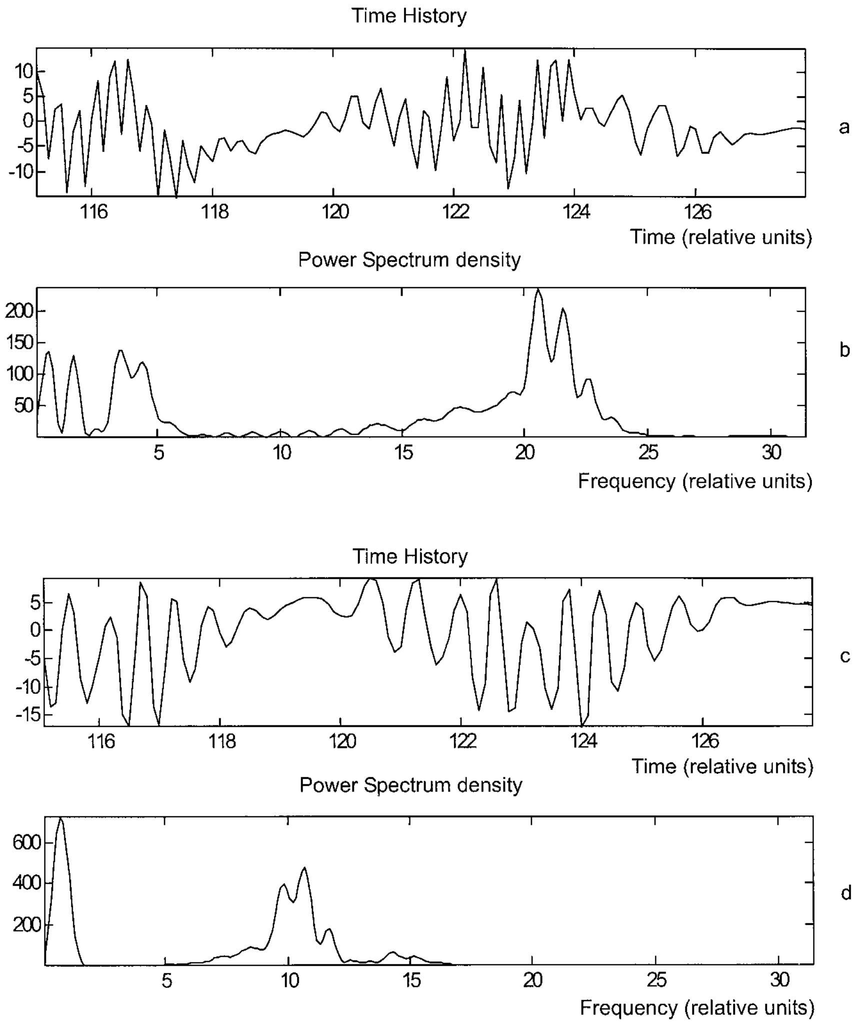

Figure 1. Solution of the decoupled systems (2) and (3) ($a = 0$): (a) coefficient α_{m2} (which is proportional to magnetic helicity in the upper layer) as a function of time in relative units; (b) power spectrum of α_{m2}; (c) real part of the total azimuthal vector-potential, $A = A_1 + A_2$, as a function of time; (d) power spectrum of A in relative units.

the high-frequency component varies considerably with the magnitude of the main sunspot cycle.

Therefore, the observed quasi-biennial variations in the magnetic helicity can be explained by the influence of the erupted low-frequency component of magnetic field on the zone of generation of the high-frequency component. This provides new evidence for the existence of the two-mode regime of the solar magnetic cycle.

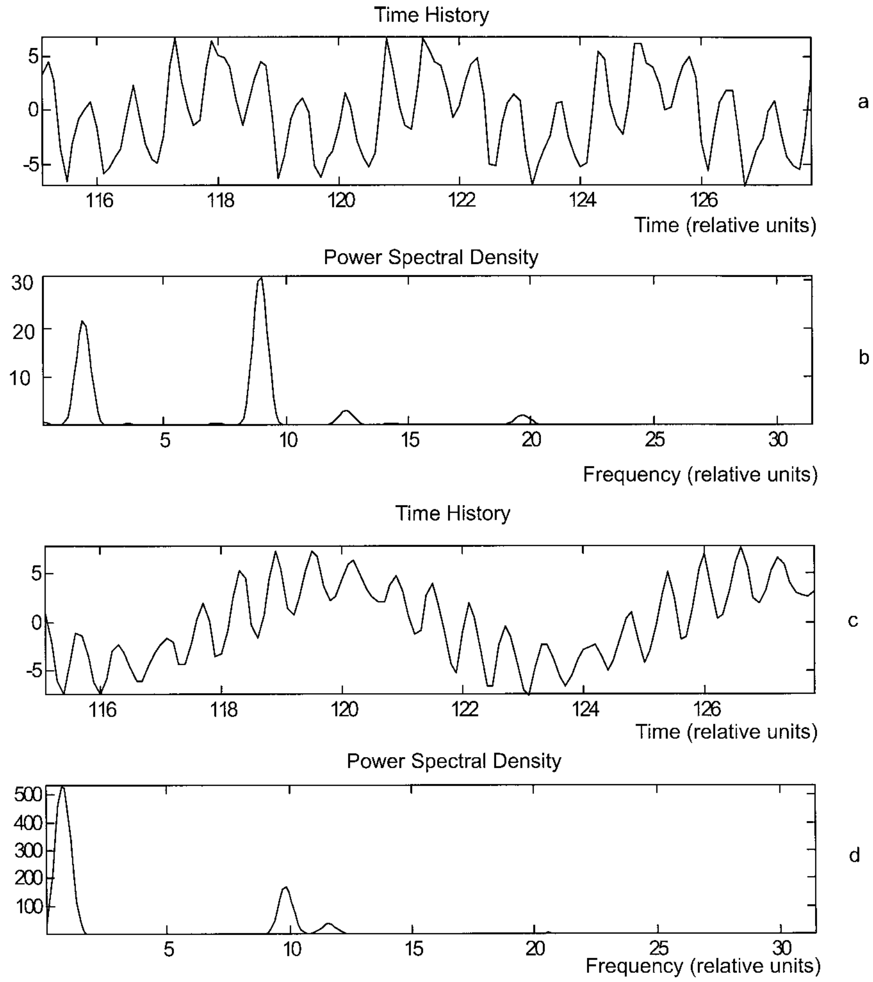

Figure 2. Case $a = 0.17$. (a) α_{m2} as function time in relative units; (b) power spectrum for α_{m2}; (c) real part of $A = A_1 + A_2$ as function time; (d) power spectrum of A in relative units.

4. Conclusions

In the frame of our phenomenological model of the double magnetic cycle, it is possible to explain the observed temporal variations in the magnetic helicity. The 11-yr and quasi-biennial periodicities are suggested to be a consequence of the influence of the erupted low-frequency magnetic field on the helicity in the near-surface region of generation of the high-frequency component of the magnetic field. In the case of a relatively weak influence, pronounced quasi-biennial components in both the magnetic field and helicity exist in the solution of the dynamo equations along with the main 11-yr period.

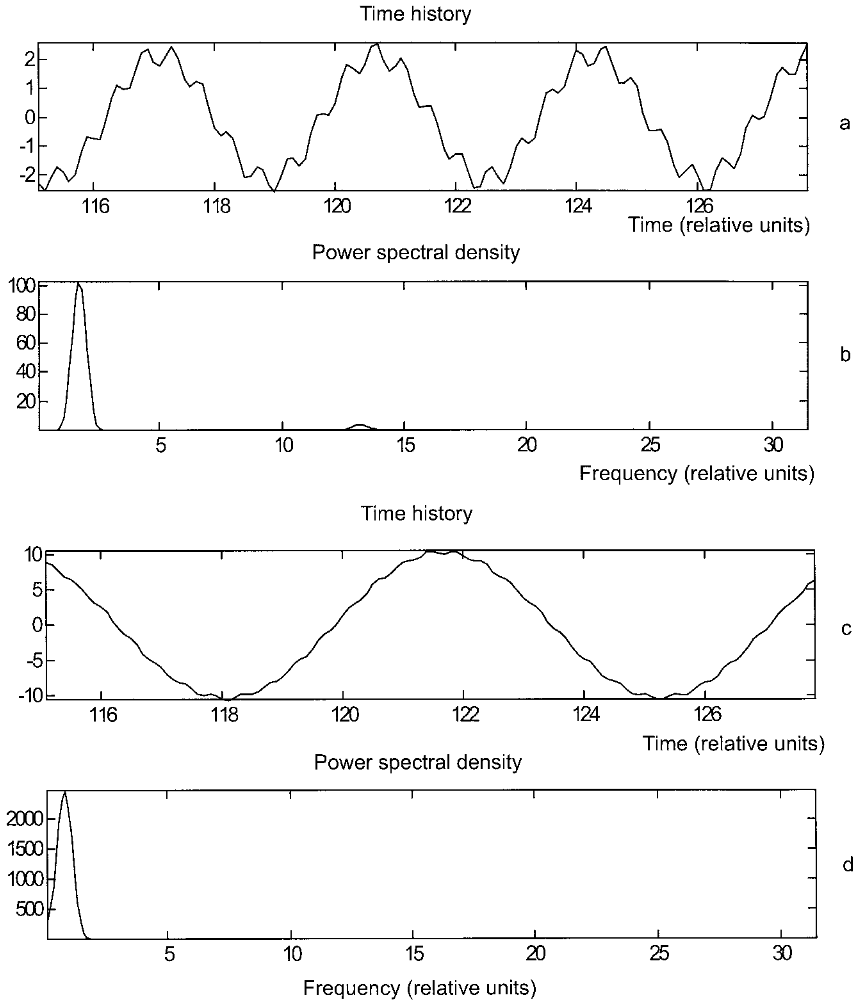

Figure 3. Case $a = 0.2$. (a) α_{m2} as function time in relative units; (b) power spectrum for α_{m2}; (c) real part of $A = A_1 + A_2$ as function time; (d) power spectrum of A in relative units.

According to our analysis of the magnetograph data (Benevolenskaya, 1996) the 2-yr component was stronger in the northern hemisphere than in the southern hemisphere in cycle 20, and vice versa in cycle 21. In cycle 22 this component was present in both hemispheres but the amplitude of the magnetic field was lower than in the previous cycles. However, the 2-year variations of the magnetic helicity were observed in cycle 22. These changes from cycle to cycle can also be explained in our model by changes in the coupling parameter a. When this parameter increases the biennial variations of the magnetic field become smaller (and more regular), but the quasi-periodic helicity variations become stronger as illustrated in Figure 2.

It will be very important for understanding the solar activity cycle to continue observations of the helicity during the current maximum of activity, and also develop numerical models of a two-component solar dynamo.

Acknowledgments

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